ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 23 МАТЕМАТИКИ Выпуск 2
2016

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A player set of *cooperative TU game* (N,ν), where $N=\{1,2,\ldots,n\}$ and $\nu: 2^N\to\mathbf{R}$, often contains the players $i,j\in N$ satisfying: $\nu(K\cup i)=\nu(K\cup j)$ for every $K\in N\setminus\{i,j\}$. Such players are called *symmetric* (ore *substitutes*). They are the participants having identical power (strength, prestige, resources, capitals) in the underlying situation. It seems natural to require that symmetric players should receive the same payoff. But the most important set-valued solution in cooperative game theory, core $C(\nu)$, does not satisfy the *equal treatment* (ET) axiom. The core allocations can assign to substitutes very different payoffs which contradicts to intuition.

The *symmetric core* $SC(\nu)$ of game (N,ν) is defined as the subset of core satisfying ET, i. e. every payoff in $SC(\nu)$ treats symmetric players equally. Denote by $\mathrm{Im}(\nu)$ the family of coalitions each of which contains only symmetric players $\Im(\nu)=\{K\in 2^N : |K|\geqslant 2,\ \textit{every}\ i,j\in K\ \textit{are substitutes}\}$. The symmetric core of symmetric ($\Im(\nu)=\{N\}$) balanced TU game is a singleton $SC(\nu)=\{(\frac{\nu(N)}{n},\ldots,\frac{\nu(N)}{n})\}$. The criterion for existence of symmetric core of such game is especially easy. It contains $n-1$ inequalities only. Symmetric core of semi-symmetric ($\Im(\nu)\neq\{N\}$, $\Im(\nu)\neq\emptyset$) game (N,ν) is not enough studied.

We have proved that balancedness of a game (N,ν) is sufficient for symmetric core existence and have described the simple non-emptiness criterion for symmetric core of n-person game with $n-1$ substitutes. It is shown that the symmetric core does not satisfy relative invariance with respect to strategic equivalenceis, so it does not provide the core axioms using modified games. Also the relation between the Lorenz solution and the symmetric core of balanced game (N,ν) is considered.

Theorem 1. *$SC(\nu)\neq\emptyset$ iff $C(\nu)\neq\emptyset$.*

Theorem 2. *Let (N,ν^0) is zero-normalized and non-negative TU game, $n\geqslant 3$, $\Im(\nu^0)=\{N\setminus\{1\}\}$,*

$$\Omega_1=\{\{2,3\},\ldots,\{2,\ldots,n\}\},\quad \Omega_2=\{\{1,2\},\ldots,\{1,\ldots,n-1\}\}.$$

Then $SC(\nu^0)\neq\emptyset$ iff the system

$$\nu^0(T)+\frac{n-|T|}{|H|}\nu^0(H)\leqslant\nu^0(N),$$

$$\frac{n-1}{|H|}\nu^0(H)\leqslant\nu^0(N),\quad H\in\Omega_1,\quad T\in\Omega_2$$

is consistent. This system contains $(n-1)(n-2)$ inequalities.

Theorem 3. *Let (N,ν^0) is zero-normalization of balanced game (N,ν) and $\Im(\nu^0)\neq\emptyset$. Then $SC(\nu^0)\subseteq SC(\nu)$ and there exist games such that $SC(\nu^0)\neq SC(\nu)$.*

Theorem 4. *Let (N,ν) is balanced game and $\Im(\nu)\neq\emptyset$. Then $SC(\nu)$ contains all Lorenz maximal core allocations and Lorenz dominates every other element of $C(\nu)$.*

Theorem 5. *Let (N,ν) is a balanced game and $\Im(\nu)\neq\emptyset$. Then*

(i) $SC(\nu)$ *does not satisfy the following core axioms: antimonotonicity, covariance and consistency (for main kinds of reduced game),*

(ii) $SC(\nu)$ *satisfies: efficiency, symmetry, modularity, reasonableness (from above) and some other core axioms based on only the original game.*

The axiomatic characterization of symmetric core is not yet provided.