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The research deals with the analysis of influence of the dependence degree between random components of an error on values of technical efficiency in stochastic frontier models. To perform the analysis several sets of data with different dependence degree between the components are modeled. The modeled data are used to obtain the estimates under two assumptions: 1) the components are independent where classical methods of estimates are used and 2) the components are dependent and this dependency is described via normal copula.

We consider the following class of 3-factor models of production function of a company: $R_i = \beta_0 L_i^{\beta_1} K_i^{\beta_2} I_i^{\beta_3} e^{V_i - U_i}$, where R_i — total production of ith company, L_i — labor input, K_i, I_i — the respective inputs of physical and intellectual capital, $V_i \sim N(0; \sigma_V^2)$, $U_i \sim N^+(\mu; \sigma_U^2)$. Random variables V_i and U_i can be statistically dependent. The data for the analysis is taken from (Aivazian et al., 2012).

Let $\rho = \rho(V_i, U_i)$ be the real value of Spearmen correlation coefficient between the error components used for modeling, $\widehat{\rho} = \widehat{\rho}(V_i, U_i)$ — its estimate, $\widehat{s} = \widehat{s}(T\widehat{E}_i, e^{-U_i})$ — estimate of the correlation coefficient between the obtained technical efficiencies $T\widehat{E}_i = \mathbf{E}\,(e^{-U_i}\,|\,V_i - U_i)$ and real values of efficiencies e^{-U_i}.

Table. Main results of the models' analysis for different values of $\rho(V_i, U_i)$

ρ values	$\rho = 0.94$		$\rho = 0.79$		$\rho = 0.39$		$\rho = 0.16$	
Compared models	M_1	M_r	M_1	M_r	M_1	M_r	M_1	M_r
$\widehat{\rho}$	0	**0.966**	0	**0.919**	0	**0.472**	0	**−0.011**
Range of varying efficiencies	(0.27; 0.95)	(**0.43**; **0.99**)	(0.33; 0.95)	(**0.45**; **0.99**)	(0.981; 0.983)	(**0.912**; **0.965**)	(0.23; 0.89)	(**0.23**; **0.89**)
$\widehat{s}(T\widehat{E}, e^{-U})$	−0.92	**0.93**	−0.64	**0.66**	−0.15	**0.13**	0.36	**0.36**
Logarithm of likelihood function	−20.78	**−19.59**	−20.81	**−20.49**	−30.42	**−29.79**	−52.11	**−52.05**

Table 1 provides for the main results obtained for the classical stochastic frontier model (M_1) and the modified model (M_r) based on normal copula. The description of the estimation method can be found in (Aivazian et al., 2014). The following conclusions can be made:

1. If the assumption regarding independence of the random error components cannot be verified one should consider their possible dependence in for obtaining estimates of technical efficiencies. Copula functions can be used to describe such dependence. Classical software can be used if there are some evidences that the error components are independent or weakly correlated.

2. Use of copula functions for consideration of possible dependence between the random components allows expanding the range of values for technical efficiencies while maintaining their ranks.

3. Specific software that provides an opportunity to use copula functions for obtaining maximum likelihood estimates needs to be developed.

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