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A. S. Fursov (Moscow, Lomonosov Moscow State University (MSU)). **Stabilization of a switched linear systems under the coordinate and parametric uncertainties.**

Key words: switched system, feedback, stabilization, control system, variable structure controller, sliding mode, linear matrix inequalities, simultaneous stabilization.

Consider switched linear system with scalar input and defined by the state equation

$$\dot{x} = (A_\sigma + \Delta A_\sigma)x + b_\sigma(u + f), \quad \sigma \in S. \tag{1}$$

where $\sigma(t)$, $\sigma : \mathbf{R}_+ \to I = \{1, 2, \ldots, m\}$ is a piecewise constant function with a finite number of discontinuities in any finite interval, S — set of such functions, called switching laws or switching signals; $x \in \mathbf{R}^n$ — state vector, $u \in \mathbf{R}$ — scalar control input; $\{A_\sigma \in \mathbf{R}^{n\times n} : \sigma \in S\}$ — family of piecewise constant matrices, $A_\sigma : \mathbf{R}_+ \to \{A_1, A_2, \ldots, A_m\}$; analogically, $\{b_\sigma \in \mathbf{R}^n : \sigma \in S\}$ — family of piecewise constant vectors, $b_\sigma : \mathbf{R}_+ \to \{b^1, \ldots, b^m\}$; $\{\Delta A_\sigma \in \mathbf{R}^{n\times n} : \sigma \in S\}$ — family of piecewise constant matrices describing parametric uncertainty of mathematical model of the switched system, $\Delta A_\sigma : \mathbf{R}_+ \to \{\Delta A_1, \Delta A_2, \ldots, \Delta A_m\}$, $\Delta A_j = b^j p^j$, where p^j ($j = 1, 2, \ldots, m$) are unknown stationary row vectors; $f(t)$ — limited external scalar continuous coordinate disturbance. Assume that relative to the vectors p^j and function $f(t)$ the estimates

$$||p^j|| \leqslant p_0 (j = 1, 2, \ldots, m), \quad |f(t)| \leqslant f_0, \quad t \geqslant 0$$

are known.

Assumption 1. $\operatorname{rank}[b_1 \ldots b_m] = 1$ and $b_2 = \lambda_2 b_1$, $b_3 = \lambda_3 b_1$, $\ldots, b_m = \lambda_m b_1$, where $\lambda_j > 0$ ($j = 2, \ldots, m$).

Problem statement. For the switched linear system (1) is required to build a stabilizing controller in the form of a discontinuous state feedback

$$u(x) = \begin{cases} u^+(x), & \text{if} \quad \rho(x) > 0, \\ u^-(x), & \text{if} \quad \rho(x) < 0 \end{cases}$$

($\rho(x) = cx$, $c \in \mathbf{R}^{1\times n}$, $u^+(x)$, $u^-(x)$ — continuous functions to choose), which

1) generates in a closed system

$$\dot{x} = (A_\sigma + \Delta A_\sigma)x + b_\sigma(u(x) + f(t)) \tag{2}$$

sliding motion on the surface $\rho(x) = 0$;

2) guarantees entering the trajectory of the system (2) to the surface $\rho(x) = 0$ from any point of the phase space;

3) ensures the exponential stability of the sliding motion of the system (2) on the surface $\rho(x) = 0$.

The equation of the sliding movement (if it exists) along the surface $\rho(x) = 0$ for a closed system (2) can be obtained using the equivalent control method. In accordance with this method, the sliding motion is given by the system of differential equations

$$\dot{z} = P_\sigma z, \quad z \in \mathbf{R}^{n-1}, \tag{3}$$

where P_σ — matrix formed by the first ($n-1$) crows and columns of the matrix

$$Q_\sigma = M^{-1}\Big(I - \frac{1}{c^\top b_\sigma} b_\sigma c^\top\Big) A_\sigma M,$$

where M — some non-singular transformation matrix.

Theorem. *Suppose switched system* (1) *satisfies assumption* 1 *and there exists a hyperplane* $\rho = 0$ $(\rho = cx)$, *along which the exponential stability of the sliding movement* (3) *is provided. Then there are coefficients* k_i $(i = 0, \ldots, n)$ *such that the variable structure controller*

$$u = -\sum_{i=1}^{n-1} k_i |x_i| \operatorname{sgn} \rho - k_n \rho - k_0 \operatorname{sgn} \rho,$$

is stabilizing for the system (1).

The question of the existence of the vector $c = (c_1, \ldots, c_n)$ reduces to the question of the existence of solutions of linear matrix inequalities, prepared in accordance with known parameters of the original switched system (1), and the coefficients k_i are calculated in accordance with the methods of the theory of simultaneous stabilization [4].

REFERENCES

1. *Fursov A. S.*, *Khusainov E. F.* On the stabilization of switchable linear systems. — Differential Equations 2015, v. 51, is. 11, p. 1518–1528.
2. *Hespanha J. P.* Uniform stability of switched linear systems: extensions of LaSalle's Invariance Principle. — Automatic Control, IEEE Transactions, 2004, v. 49, № 4, p. 470–482.
3. *Liberzon D.*, *Morse A. S.* Basic problems in stability and design of switched systems. — IEEE Control Systems, 1999, v. 19, № 5, p. 59–70.
4. *Emel'yanov S. V.*, *Fomichev V. V.*, *Fursov A. S.* Simultaneous stabilization of linear dynamic plants by the variable-structure controller. — Automation and Remote Control, 2012, v. 73, is. 7, p. 1126–1133.