II INTERNATIONAL BALTIC SYMPOSIUM ON APPLIED AND INDUSTRIAL MATHEMATICS

V. V. Fomichev (Moscow, Lomonosov Moscow State University (MSU)). **About the properties of relative degree for MIMO systems.**

For the modern control theory is an important concept of zero dynamics and relative degree. For stationary linear SISO systems, these concepts are well known. However, for MIMO systems the definition of the relative degree is a problem.

Consider the linear time-independent dynamical system

$$\dot{x}(t) = Ax(t) + B\xi(t), \quad y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $y(t) \in \mathbf{R}^m$, and $\xi(t) \in \mathbf{R}^m$, $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times l}$ and $C \in \mathbf{R}^{l\times n}$.

Next, we believe that this system is controllable and observable. For this system, the following definition of relative degree (on Isidori) can be given.

Definition 1. A vector $r = (r_1, r_2, \ldots, r_l)$ is called the vector of *relative degree* (the RD vector) of system (1) if the following conditions are satisfied.

1. $C_iB = 0$, $C_iAB = 0, \ldots, C_iA^{r_i-2}B = 0$, and $C_iA^{r_i-1}B \neq 0$.
2. $$\det H(r_1, \ldots, r_l) = \det \begin{pmatrix} C_1A^{r_1-1}B \\ \cdots \\ C_lA^{r_l-1}B \end{pmatrix} \neq 0.$$

Here the C_i, $i = 1, 2, \ldots, l$, are the rows of the matrix C. It was shown in the monograph [2, p. 72–82] and [1] that conditions 1 and 2 can be inconsistent.

In some cases, nonsingular transformation of the outputs to ensure that the conditions of the definition were compatible. Such a transformation does not always exist. In [3] we give the necessary and sufficient conditions under which such a transformation exists. Let's give the following definitions.

Definition 2. If only the first condition in Definition 1 holds for a vector r, then we call it the vector of *incomplete relative degree* (IRD vector).

Let us arrange the components of the IRD vector in ascending order. (This is always possible by an appropriate renumbering of rows of the matrix C.) Suppose that there are k distinct components of that vector. Then

$$r = (r_1^{(1)}, r_2^{(1)}, \ldots, r_{n_1}^{(1)}, \ldots, r_1^{(s)}, r_2^{(s)}, \ldots, r_{n_s}^{(s)}, \ldots, r_1^{(k)}, r_2^{(k)}, \ldots, r_{n_k}^{(k)}),$$

where $r_i^{(p)} = r_j^{(p)}$, $i, j \in \{1, 2, \ldots, n_p\}$; $r_i^{(p)} < r_j^{(q)}$ if $p < q$, $i \in \{1, 2, \ldots, n_p\}$, $j \in \{1, 2, \ldots, n_q\}$, $n_1 + n_2 + \cdots + n_k = l$, i.e., the vector r splits into "sections" consisting of identical elements.

Condition 2 in Definition 1 essentially deals with the linear independence of the rows $H_m^{(p)}$ for $p = 1, \ldots, k$ and $m = 1, \ldots, n_p$. We weaken this condition and consider the following definition.

Definition 3. A vector r is called a leading IRD (LIRD) of system (1) if r is an IRD vector and the rows $H_j^{(p)}{}_{j=1}^{n_p}$ are linearly independent for all $p \in \{1, 2, \ldots, k\}$. In other words, r is an LIRD vector if the rows $H_m^{(p)}$ are linearly independent inside each section; the linear independence of the entire set of rows is not required.

Any controllable and observable system can be reduced to the form with LIRD. There exists constructive algorithm to transform the system to this kind.

By using an LIRD vector, one can determine whether a system can be reduced by a nonsingular linear transformation of outputs to a form with an RD vector. This is a consequence of the following assertion.

Theorem. *A system* (1) *cannot be reduced* [3] *by a nonsingular transformation of outputs to a form with an Isidori* RD *if and only if its* LIRD *vector* r *is not an Isidori* LIRD *vector.*

REFERENCES

1. *Korovin S. K., Il'in A. V., and Fomichev V. V.* Zero Dynamics for Linear Vector Time-Invariant Systems. — Dokl. Akad. Nauk Teor. Upravl., 2007, v. 414, № 5, p. 598–604.
2. *Il'in A. V., Korovin S. K., Fomichev V. V.* Metody robastnogo obrashcheniya dinamicheskikh sistem (Method of Robust Inversion for Dynamical Systems), Moscow, 2009.
3. *Kraev A. V., Rogovskii A. I., Fomichev V. V.* Differential Equations, 2014, v. 50, № 8, p. 1122–1127.