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L. M i r o n o v a (Moscow, MGMU (MAMI)). **Model problem for estimation of thermostressed condition of welded structural members.**

We can determine the estimation parameters of optimal state for a structural component under heterogeneous high-gradient thermal effect, if we compare them with the maximum or minimum effect (extremum) on this state [1].

Let a free infinite isotropic cylindrical shell of radius R with the following specific parameters: $A = 1$, $R_1 \to \infty$, $B = R_2 = R$, $k_1 = 0$, $k_2 = 1/R$, $2h \ll R$, (Fig.) is affected by a symmetric temperature field of constant thickness and vanishing at infinity. Boundary conditions are defined as follows: $\frac{\partial t}{\partial \gamma} = 0$; $T^{+}_{(x)} - T^{-}_{(x)} = \Delta T^{\pm}_{(x)} = \frac{\lambda}{c_p \rho} 2h$.

Find the extreme temperature field and determine the correspondent deflected mode of a shell. This problem belongs to the class of extremum problems and reduces to finding an extremum of elastic strain energy functional of a shell at the set of movement functions u, v, w and temperature strain and moment which satisfy a system of equations, conditions of end section fixing and some of additional constraint conditions.

Thermoelastic condition of a shell is characterized by circumferential force N and axial moment M, which are determined by temperature and represented in the following way:

$$N = 2Eh(w_0 - \alpha T), \quad M = -\frac{EhR}{2a^2} \cdot \frac{R^2}{a^2} \frac{d^2 w_0}{dz^2},$$

where coefficient a identifies geometric parameters of the shell and is determined by relation $a = [(1 - v^2)\frac{3R^2}{4h^2}]^{1/4}$; where $2h$ is a shell thickness, E is elastic modulus, v is Poisson's ratio; α is linear coefficient of thermal expansion; z is an axial coordinate; w_0 is a dimensionless deflection; $w_0 = w/R$; T is an average temperature characteristic of the temperature field.

Function of deflection w_0 should satisfy the resolving equation [3]

$$\Big(\frac{d^4 w_0}{dz^4}\Big) + 4(w_0 - \alpha T_1) = 0.$$

The elastic energy of a shell is a functional determined at the set of functions $w_0 = w_0(z)$ and represented by the relation [4]

$$K[w_0] = \frac{\pi E h R^2}{8a} \int_{-\infty}^{+\infty} \Big[\Big(\frac{d^4 w_0}{dz^4}\Big)^2 + 4\Big(\frac{d^2 w_0}{dz^2}\Big)^2\Big] dz.$$

Let's formulate the following variational problem. Find an extremum of functional $K[w_0]$ at the set of such vanishing at infinity functions $w_0 = w_0(z)$ which are continuous with first, second and third continuous derivatives, and which satisfy the following conditions in the fixed sections $z = z_j$ $(j = 1, 2, \ldots, n)$

$$\frac{d^{(K)} w(z_j)}{dz^K} = w_{kj} \quad (K = 0, 1, 2, 3),$$

where w_{kj} are arbitrary numbers which can be found if we give numeric parameters of the problem (deflection, temperature, circumferential force, axial moment, stress).

When the whole system of equations is solved, including Euler equations and conditions at infinity, fixing of end sections and additional nonholonomic constraints, we get the following family of extreme temperature fields

$$T = \frac{c_1}{2c}\sqrt{\frac{h}{R}} \cdot T_0\left[cy_0 + e^{-cy_0}\big((1+cy_0)\cos cy_0 - \sin cY_0\big)\right],$$

where $c = \sqrt[4]{\frac{2}{1-v}}$; $c_1 = \sqrt{\frac{2}{3(1-v^2)}}$; y_0 is a linear coordinate of the cross section arc (β) of cylindrical shell, T_0 is the given temperature of the considered cross section at local heating.

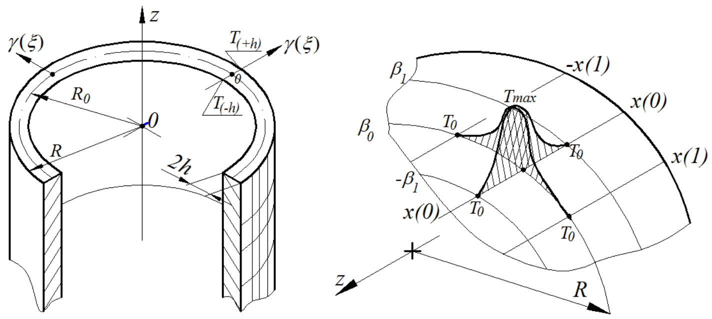

Fig. Computational model of the extreme temperature field

Then, thermal stresses are determined according to the formula:

$$\sigma_\beta = \pm\frac{E\alpha\Delta T}{2(1-\nu)}, \quad \Delta T = \frac{\lambda}{c_p\rho}2h,$$

where λ is a heat conductivity coefficient; c_p is a specific heat capacity of material; ρ — is material density; $2h$ is a thickness of shell.

This model problem is perfectly acceptable for determination of residual stresses in welded shell structures, for example pressure vessels.

REFERENCES

1. *Mironova L. I.* Parameters of optimum control of the thermotension of designs of obolochechnogo of type at thermal loading. — Engineering & Automation Problems. 2013, № 1, p. 101–105. (In Russ.).