ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 23 МАТЕМАТИКИ Выпуск 2
2016

A. A. Kudryavtsev, O. V. Shestakov (Moscow, Lomonosov Moscow State University, Institute of Informatics Problems, Federal Research Center “Computer Science and Control” of the Russian Academy of Sciences). **An optimum threshold minimizing the probability-of-error criterion.**

Estimating a signal function from the noisy observations using wavelet methods is a problem that has been drawing a great attention over the last decades. The basic idea behind the wavelet estimation is to get a relatively small number of wavelet coefficients to represent the underlying signal function. A value called the threshold is used to remove or keep the wavelet coefficient. Hence, estimation quality depends on how efficient threshold value would be chosen [1].

We consider the following data model:

$$X_i = f_i + w_i, \quad i = 1, \ldots, 2^J,$$

where f_i are “clean” samples of the signal function from the Lipschitz class $\mathrm{Lip}(\gamma)$ with $\gamma > 0$ and $w_i \sim N(0, \sigma^2)$ are samples from a white Gaussian noise. Applying the wavelet transform we obtain the following model of the empirical wavelet coefficients:

$$Y_{j,k} = \mu_{j,k} + W_{j,k}, \quad j = 0, 1, \ldots, J-1, \quad k = 0, 1, \ldots, 2^j - 1,$$

where $W_{j,k}$ have the same statistical structure as w_i.

Denote by $\widehat{Y}_{j,k}$ the estimate of the wavelet coefficient which is obtained with the use of the soft thresholding function $\rho_T^{(s)}(x) = \mathrm{sign}(x)(|x| - T)_+$ or the hard thresholding function $\rho_T^{(h)}(x) = x\,\mathbf{1}\{|x| > T\}$ and the threshold value T.

To find an optimum threshold we consider the cost function

$$R_J = \sup_{f \in \mathrm{Lip}(\gamma)} \mathbf{P}\left\{ \max_{j,k} \left| \widehat{Y}_{j,k} - \mu_{j,k} \right| > \varepsilon \right\}$$

with a given critical value $\varepsilon > 0$. The function R_J is a generalization of the cost function proposed in [2]. As the number of samples grows, R_J tends to 1. Our goal is to find an optimum threshold ensuring the minimum loss in the sense that the rate of convergence of R_J to 1 is the slowest.

For the soft thresholding method we have the following statement.

Theorem. *The optimum soft threshold minimizing the rate of convergence of the cost function R_J to 1 behaves asymptotically as*

$$T_*^{(s)} \sim \sigma \sqrt{\frac{4\gamma}{2\gamma + 1} \log 2^J} - \varepsilon.$$

It appears that for the hard thresholding method $R_J = 1$ starting with some J. This fact means that there is no sense in estimating the cost function R_J when using the hard thresholding function. It is an interesting observation since this is not the case when estimating the mean squared error [3].

Acknowledgements. This research is supported by Russian Scientific Foundation, project 14-11-00364.

REFERENCES

1. *Jansen M.* Noise Reduction by Wavelet Thresholding. Heidelberg–N. Y. etc.: Springer, 2001, 224 p. (Ser Lect. Notes Statist. V. 161.)
2. *Sadasivan J.*, *Mukherjee S.*, *Seelamantula C. S.* An Optimum Shrinkage Estimator Based on Minimum-Probability-Of-Error Criterion and Application to Signal Denoising. — In: 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP, Florence, May 4–9, 2014). Piscataway, NJ: IEEE, p. 4249–4253.
3. *Markin A. V.*, *Shestakov O. V.* Consistency of risk estimation with thresholding of wavelet coefficients. — Moscow Univ. Comput. Math. Cybernet., 2010, v. 34, is. 1, p. 22–30.