XXXIII INTERNATIONAL SEMINAR ON STABILITY PROBLEMS FOR STOCHASTIC MODELS

S. Nagaev, V. Chebotarev[1] (Sobolev Institute of Mathematics, Novosibirck, Russia, Computing Centre FEB RAS, Russia). **On bounds for large deviations probabilities for the Binomial distribution.**

Denote by F the distribution function of the binomial law with parameters n and $0<p<1$. We shall use the following notations:

$$q=1-p, \quad H(q,1-x)=(1-x)\ln\frac{1-x}{q}+x\ln\frac{x}{1-q}, \quad \varphi(t)=\frac{1}{\sqrt{2\pi}}\,e^{-t^2/2},$$

$$\Phi(x)=\int_{-\infty}^{x}\varphi(t)\,dt, \quad A(x,n,p)=\frac{1}{\sqrt{2\pi}}\,M\Big(\sqrt{nx(1-x)}\,\ln\frac{xq}{(1-x)p}\Big),$$

where $0<x<1$, $M(t):=(1-\Phi(t))/\varphi(t)$ is the Mills ratio, and let

$$\beta_3(y)=\frac{y^2+(1-y)^2}{\sqrt{y(1-y)}}.$$

Now we formulate the statements proved in the work.

Theorem 1. *If* $0<p<x<1$, *then*

$$\Big|1-F(nx)-A(x,n,p)\,e^{-nH(q,1-x)}\Big|\leqslant\frac{0.82}{\sqrt{n}}\,\beta_3(x)\,e^{-nH(q,1-x)}. \tag{1}$$

Theorem 1 entails

Corollary 1. 1) *If* $0<p<x<1$, *then*

$$\Big(A(x,n,p)-\frac{0.82}{\sqrt{n}}\,\beta_3(x)\Big)\,e^{-nH(q,1-x)}$$
$$\leqslant 1-F(nx)\leqslant\Big(A(x,n,p)+\frac{0.82}{\sqrt{n}}\,\beta_3(x)\Big)\,e^{-nH(q,1-x)}.$$

2) *If* $x-p=O(\frac{1}{\sqrt{n}})$ *as* $n\to\infty$, *then*

$$A(x,n,p)\,e^{-nH(q,1-x)}\sim 1-\Phi\Big(\frac{(x-p)\sqrt{n}}{\sqrt{pq}}\Big),$$

$$\beta_3(x)\,e^{-nH(q,1-x)}\sim\sqrt{2\pi}\,\beta_3(p)\,\varphi\Big(\frac{(x-p)\sqrt{n}}{\sqrt{pq}}\Big).$$

Corollary 2. *Let* $X, X_1,\dots,X_n$ *be independent random variables with one and the same two-point distribution*: $\mathbf{P}\{X=a\}=p$, $\mathbf{P}\{X=b\}=q$, $a>b$. *Then for all* y, *satisfying the condition* $\mathbf{E}\,X<y<a$, *the following inequality holds*:

$$\Big|\mathbf{P}\Big\{\sum_{j=1}^{n}X_j\geqslant ny\Big\}-e^{-n\widetilde{H}}A\Big(\frac{y-b}{a-b},p,n\Big)\Big|\leqslant\frac{0.82}{\sqrt{n}}\,\beta_3\Big(\frac{y-b}{a-b}\Big)e^{-n\widetilde{H}}, \qquad \widetilde{H}=H\Big(q,\frac{a-y}{a-b}\Big).$$

We can show that under condition $\sqrt{nx(1-x)}\geqslant 1.64$ the inequality (1) is more accurate with respect to the Chernov – Höffding bound $1-F(nx)\leqslant e^{-nH(q,1-x)}$.

Let us consider the Poisson approximation.

Denote by $\Pi_\lambda(\cdot)$ the distribution function of the Poisson law with a parameter λ. We shall use the notation $\lambda_1=np(1-x)/q$.

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Theorem 2. *If* $0<p<x<1$, *then*

$$1-F(nx)=\Big(\frac{q}{1-x}\Big)^n e^{-n\,(x-p)/q}\Big(1-\Pi_{\lambda_1}(nx)\Big)+R,$$

where $|R|\leqslant 2e^{-n\,H(q,1-x)}\min\{x,nx^2\}$, *and*

$$\Big(\frac{q}{1-x}\Big)^n e^{-n\,(x-p)/q}\sim e^{-n\,p\,(x-p)/q},\qquad x\to 0.$$

Theorem 3. *If* $0<p<x<1$, *and the conditions* $nx\geqslant 1$, $x\leqslant\delta_1<1$, $p/x\leqslant\delta_2<1$ *are fulfilled, then*

$$\begin{aligned}&1-F(nx)\\&=\Big(1-\Pi_{np}(nx)\Big)\Big(\frac{1-x}{q}\Big)^{\lceil nx\rceil-n}e^{-n\,(x-p)/q}\Big(1+\frac{\theta_1\,np(x-p)}{q\,e^{n\,p\,(x-p)/q}}\Big)\Big(1+\frac{\theta_2\,p}{x\,(1-\delta_2)}\Big)\\&\quad+2\,\theta_3\,e^{-n\,H(q,1-x)}\,x,\end{aligned}$$

where $0<\theta_1<1$, $|\theta_i|<1$, $i=2,3$.

Recall that $\lceil a\rceil$ is the smallest integer satisfying the inequality $a\leqslant\lceil a\rceil$.

A comparison with the results of [1–6] is provided.

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