XXXIII INTERNATIONAL SEMINAR ON STABILITY PROBLEMS FOR STOCHASTIC MODELS

S. Nagaev [1] (Sobolev Institute of Mathematics, Novosibirsk, Russia). **The Berry–Esseen bounds for general Markov chains.**

Let $\{X_n\}_{n=0}^{\infty}$ be a homogenious Markov chain with a transition function $p(x,B), x\in\mathcal{X}$, $B\subset\mathbf{S}$, where $(\mathcal{X},\mathbf{S})$ is a measurable space. Suppose that $\{X_n\}_{n=0}^{\infty}$ satisfies the next two conditions:

a) there exists the set $A_0\subset\mathbf{S}$, nonnegative measure φ on $A_0\subset\mathbf{S}$ with $\varphi(A_0)>0$ and $n_0\geqslant 1$ such that $p^{(n_0)}(x,B)\geqslant\varphi(B)$ for all $x\in A_0$, $B\subset A_0\subset\mathbf{S}$;

b) $\mathbf{P}_x\bigl\{\bigcup_{n=1}^{\infty}\{X_n\in A_0\}\bigr\}=1$ for every $x\in\mathcal{X}$.

Here and in what follows $\mathbf{P}_x$ denotes the probability on the space of trajectories of the process $\bigl\{X_n\bigr\}_{n=0}^{\infty}$ under the condition $X_0=x$.

Define the submarkovian transition function $\varphi(\cdot,\cdot)$ by the equalities $\varphi(x,B)=\varphi(A_0B)$ for $x\in A_0$ and $\varphi(x,B)=0$ for $x\in\mathcal{X}\setminus A_0$. Let $w(\cdot,\cdot)=p(\cdot,\cdot)-\varphi(\cdot,\cdot)$ and $w^{(k)}(\cdot,\cdot)$ be k th iteration of $w(\cdot,\cdot)$. Denote $q(x,\cdot)=\sum_{k=0}^{\infty}w^{(k)}(x,\cdot)$, where $w^{(0)}(x,\cdot)$ is a measure concentrated in x. The measure

$$q(\cdot)=\int_{A_0}q(x,\cdot)\,\varphi(dx)$$

is invariant for the process $\{X_n\}$ (see, e. g., [1]). Let us add to $\mathcal{X}$ a new element, say, x^0. Denote this extension by $\mathcal{X}^0$. Let $\mathbf{S}^0$ be the least σ-algebra containing $\mathbf{S}\cup x^0$. Introduce the transition function on $(\mathcal{X}^0,\mathbf{S}^0)$ letting $p_0(x,\{x^0\})=1$ for $x\in A_0$, $p_0(x,\{x^0\})=0$ for $x\in\mathcal{X}\setminus A_0$, $p_0(x,B)=p(x,B)$ if $x\in\mathcal{X}$, $B\in\mathbf{S}$. Let $\bigl\{X_n^0\bigr\}_{n=0}^{\infty}$ be the Markov chain corresponding to the transition function $p_0(\cdot,\cdot)$. Denote

$$\alpha_s=\sum_{n=0}^{\infty}\int_{A_0}\varphi(dx)\,\mathbf{E}_x\biggl\{\Bigl|\sum_{k=0}^{n}g(X_k^0)\Bigr|^s;X_n^0\in A_0,\ N>n\biggr\}.$$

Here $g(x), x\in\mathcal{X}$, is a real function which is measurable with respect to $\mathbf{S}$, $N=\min\{n>0: X_n\in\mathcal{X}_0\}$. Let

$$p_k=\int_{A_0}w^{(k-1)}(x,A_0)\,\varphi(dx),\quad \beta_s=\sum_{k=1}^{\infty}k^s p_k,\quad \beta_s(x)=\sum_{k=0}^{\infty}(k+1)^s\,w^k(x,A_0).$$

Further, define

$$a_0=\int_{\mathcal{X}}g(x)\,q(dx),\quad b^2=\int_{\mathcal{X}}g^2(x)\,q(dx),\quad m(x)=\int_{\mathcal{X}}|g(y)|\,q(x,dy).$$

Theorem. *Let* $n_0=1$, b^2, $m(x)$, α_3, β_3, $\beta_2(x)$ *be finite,* $\sigma^2>0$, $a_0=0$. *Then*

$$\begin{aligned}\sup_r&\left|\mathbf{P}_x\Bigl\{\frac{1}{\sqrt{n}}\sum_{k=1}^{n}g(X_k)<r\Bigr\}-\Phi\Bigl(\frac{r\sqrt{\mu}}{\sigma}\Bigr)\right|\\ &<c\left[\frac{1}{\sqrt{n}}\Bigl(\frac{\alpha_3\sqrt{\mu}}{\sigma^3}+\frac{\beta_3\,\mu(x)}{\mu^{5/2}}+\frac{b}{\sigma}\sqrt{\frac{\beta_3}{\mu}}\,\mu(x)+\frac{\beta_2(x)\sqrt{\mu}}{\beta_2}+\frac{m(x)\sqrt{\mu}}{\sigma\varphi(A_0)}\Bigr)\right.\\ &\qquad\left.+\frac{\ln n}{\mu n}\Bigl(\frac{\beta_2}{\varphi(A_0)}\Bigr)^2\right],\end{aligned}$$

where Φ *is the standard normal law,* c *is an absolute constant,* $\sigma^2=\alpha_2$, $\mu=\beta_1$, $\mu(x)=\beta_1(x)$.

[1] e-mail: nagaevmath.nsc.ru

Bounds $O(n^{-1/2})$ were obtained in [2, 3], however, without explicit expressions for the dependency of parameters of the chain. The authors of cited works applied the so-called splitting method introduced in [4, 5]. By contrast, the method of proving the above-stated theorem is pure analytic.

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