ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 23 МАТЕМАТИКИ Выпуск 2
2016

S. Melnikov (Moscow, «Linfo»). **Stationary distribution of random walk on the generalized de Bruijn digraphs.**

Let n, d be two positive integers. The generalized de Bruijn digraph ([1, 2]) have vertices, labeled by integers modulo n. The vertex i is adjacent to vertices $di+r(\bmod n)$, $0 \leqslant i \leqslant n-1$, $0 \leqslant r \leqslant d-1$. $G(n,d)$ is regular of degree d and have nd arcs. Digraph $G(d^t,d)$ is classical d-ary de Bruijn graph of degree t, $t=1,2,\ldots$.

Let's consider a nearest-neighbor random walk on $G(n,d)$. It starts from a randomly chosen vertex and steps along arcs into one of the neighboring vertices. This random walk can be associated with a sequence of independent and identically distributed random numbers modulo d. Let p_r denote the probability of the step from vertex i to vertex $di+r(\bmod n)$, $\sum_{r=0}^{d-1} p_r = 1$, $0 < p_r < 1$.

Given a random walk, the important question is to determine the stationary distribution, which intuitively is the state that is reached after taking many steps in the random walk. Some problems connected with the random walks on the classical de Bruijn graphs are studied, e. g., in [3, 4] ($p_r = 1/d$ case) and [5, 6] (general p_r case).

Theorem. Let $n = sd^t$, $t \geqslant 0$, $GCD(s,d) = 1$. For $0 \leqslant q \leqslant n-1$ let $b_r(q)$ be the number of occurrences "r" in the list of the base d digits in the integer $q \bmod (d^t)$. Stationary distribution of random walk on $G(n,d)$ is:

$$P(q) = \frac{1}{s}\prod_{r=0}^{d-1} p_r^{b_r(q)}, \qquad 0 \leqslant q \leqslant n-1.$$

Corollary. *If* $GCD(n,d)=1$, *then stationary distribution is uniform and does not depend on variables* $\{p_r, 0 \leqslant r < d\}$:

$$P(q) = \frac{1}{n}, \qquad 0 \leqslant q \leqslant n-1.$$

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