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N. D. Leontyev [1] (Moscow, Lomonosov Moscow State University). **Analysis of a queueing system with autoregressive arrivals.**

Methods of evaluation of traffic intensity play a very important role in the development of complex information systems as they enable to detect insufficient or excess computing resources on the basis of respective models. For queueing systems with batch arrivals traffic intensity may be modeled by an autoregressive relationship between batch sizes.

Systems with autoregressive parameters are considered in many studies on queueing theory. Let us mention the papers [1] and [2], which study a queueing system with discrete time, where batch sizes exhibit autoregressive relation.

Following the paper [3], we consider a single-server queueing system with batch Poisson arrival process with intensity a. Suppose that the system has infinite capacity, and service time has a distribution $B(x)$ with density $b(x)$ and Laplace transform $\beta(s)$ of this density. Incoming batches can have sizes $1,2,\ldots,M$ with the corresponding probabilities $h_1,h_2,\ldots,h_M$. Moreover, the size of the nth batch either equals the size of $(n-1)$st batch with probability $0\leqslant p<1$, or is an independent random variable with probability $1-p$.

Let's define the following stochastic processes:

$L(t)$ — queue length at time t;

$X(t)$ — elapsed time from the start of service of the customer being served at t;

$N(t)$ — size of the latest batch arrived before t.

Let

$$P(n,k,x,t) \;=\; \frac{\partial}{\partial x}\,\mathbf{P}\bigl\{L(t)=n,\ N(t)=k,\ X(t)<x\bigr\},$$

$$P(n,k,t) \;=\; \mathbf{P}\bigl\{L(t)=n,\ N(t)=k\bigr\},$$

and

$$\pi(z,k,x,s) \;=\; \sum_{n=1}^{\infty} z^n \int_0^{\infty} e^{-s\,t} P(n,k,x,t)\,dt,$$

$$\pi_0(k,s) \;=\; \int_0^{\infty} e^{-s\,t} P(0,k,t)\,dt$$

for $|\,z\,|\leqslant 1$, $\mathrm{Re}(s)>0$.

The following theorem gives a formula to determine the queue length distribution at an arbitrary moment.

Theorem. *Function* $\pi(z,k,x,s)$ *for* $|\,z\,|<1$, $k=1,2,\ldots,M$, $x\geqslant 0$, $\mathrm{Re}(s)>0$ *is determined by the expression*

$$\pi(z,k,x,s)=\bigl(1-B(x)\bigr)\sum_{n=1}^{M} C_n(z,s)\,\frac{(1-p)\,h_k a\,z^k}{\widetilde{\lambda}_n(z)-p\,a\,z^k}\,\exp\bigl\{-\bigl(s+a-\widetilde{\lambda}_n(z)\bigr)\,x\bigr\},$$

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where

$$C_n(z,s) \;=\; \frac{1}{1-\beta\,(s+a-\widetilde{\lambda}_n(z))/z}\; \frac{\prod_{i=1}^{M}\big(\widetilde{\lambda}_n(z)-p\,a\,z^i\big)}{\prod_{\substack{j=1\\ j\neq n}}^{M}\big(\widetilde{\lambda}_n(z)-\widetilde{\lambda}_j(z)\big)} \sum_{m=1}^{M}\frac{b_m(z,s)}{\widetilde{\lambda}_n(z)-p\,a\,z^m},$$

$$b_m(z,s) \;=\; -(s+a)\,\pi_0(m,s)+h_m+\Big[p\,\pi_0(m,s)\,a\,z^m+(1-p)\sum_{n=1}^{M}\pi_0(n,s)\,h_m\,a\,z^m\Big],$$

parameters $\widetilde{\lambda}_1(z),\widetilde{\lambda}_2(z),\ldots,\widetilde{\lambda}_M(z)$ *are determined from the equation*

$$\prod_{i=1}^{M}(p\,a\,z^i-\widetilde{\lambda})+\sum_{i=1}^{M}(1-p)\,h_i a\,z^i\prod_{\substack{j=1\\ j\neq i}}^{M}(p\,a\,z^j-\widetilde{\lambda})=0,$$

and $\pi_0(1,s),\pi_0(2,s),\ldots,\pi_0(M,s)$ — *from the system*

$$\sum_{m=1}^{M}\prod_{\substack{j=1\\ j\neq m}}^{M}\big(\widetilde{\lambda}_n(z_n)-p\,a\,z_n^j\big)\big(-(s+a)\,\pi_0(m,s)+h_m+\widetilde{\lambda}_n(z_n)\pi_0(m,s)\big)=0,$$

where $z_n=z_n(s)$ *is the solution of the functional equation* $z_n=\beta\,(s+a-\widetilde{\lambda}_n(z_n))$.

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