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A. V. B u l i n s k i (Moscow, MIPT)**. Employment of operator algebras in quantum information theory for analysis of infinite-dimensional systems stability problems.**

The concepts and methods of quantum information theory as well as the majority of fruitful results achieved by the end of the first decade of this century were focused on analysis of finite-dimensional quantum systems. The system states, observables and operations on them were described in terms of (matrix analysis of) operators on finite-dimensional spaces, see, e. g., [1].

We are interested in passing to the case of infinite-dimensional separable Hilbert spaces $\mathcal{H}$ and, moreover, in possibility to consider the system states represented not only by states of the whole operator algebra $\mathcal{B}(\mathcal{H})$ but also by ones on other von Neumann algebras appearing in quantum statistical mechanics and quantum field theory. The reason is to study models involving bosonic and fermionic quantum fields as resources for solving information tasks.

Notably the information encoded in quantum correlations is typically fragile and the quantum channels processing and transmitting it are ubiquitously noisy. In other words the evolution of a system non-isolated from environment is accompanied by dissipation, decoherence and dephasing. The problems of stability of quantum channels performance are diverse, some of them are related to decoding and error-correction.

In this contribution, we return to the problem of characterizing the (decoding) measurement maps, or quantum-classical (QC), channels Λ , see [1]. Their action on a bipartite quantum system results in vanishing of (one-sided) quantum discord. However, one has to consider also related multipartite systems. As was established in [2], description of such channels partially destroying quantum correlations is connected with the problem of local broadcasting (i. e. a kind of reproduction of a part) of bipartite state. Fortunately, one can equivalently investigate only local broadcasting of correlations within a bipartite state. Such description involves analysis of short quantum Markov chains, otherwise, tripartite states with vanishing certain conditional mutual information. On the other hand, a characterization in terms of quantum-classicality of the Choi-Jamiolkowski matrix corresponding to Λ was given. Moreover, an intrinsic classical Markov chain emerges and its ergodic properties lead to inference on asymptotic behavior of iterates Λ^n, $n \in \mathbb{N}$. The systems were assumed finite-dimensional. We will deal with several lines of extension of the mentioned results. Recall that asymptotic behavior of semigroup of iterates of a quantum operation (channel) was tackled in [3] in the context of dynamical systems formed by quantum Markov semigroups on general operator algebras in a scenario of emergence of decoherence-free subalgebra.

Treating a quantum channel (operation in Heisenberg picture) as a normal unital completely positive linear map between von Neumann algebras we establish

Theorem 1. *The classes of QC-channels as well as entanglement-breaking and entanglement-saving ES-channels can be introduced for the injective von Neumann algebras $\mathcal{M}$ in separable H-spaces.*

These channels retain the main properties of finite-dimensional analogs, for ES-channels see [4] . We take into account the stabilization of finite-dimensional approximations to continuous measurement modeled on von Neumann algebra in [5]. Furthermore, the appropriate information measures are needed in more general algebraic setting [6]. Due to the recent trend of employing measures based on various quantum generalizations of classical Renyi entropies instead of von Neumann quantum analog of classical Shannon entropy, in the vein of [7] we verify whether characteristics of measuring channels are intact to the change of basic entropy.

In this respect note the quite recent paper [8] on the equality attainment in the data processing inequality for generalized relative entropy measure on states of $B(\mathcal{H})$, $\dim\mathcal{H} < \infty$.

Lemma 1. *The algebraic characterization in* [8] *of equality attainment in the data processing inequality based on sandwiched Rényi divergence carries over to infinite-dimensional case of* $B(\mathcal{H})$ *and any semi-finite von Neumann algebra* $\mathcal{M}$*. Thus it turns possible to introduce the appropriate recovery channel associated to an* (*approximate*) *short quantum Markov chain.*

Lemma 2. *The notion of fidelity extends to the states of a semi-finite algebra.*

The stability of approximate Markov property was characterized in [9], with a bound in terms of fidelity on approximate equality in sort of data processing inequality.

Theorem 2. *The Fawzi-Renner estimate has a generalization to states on a semi-finite von Neumann algebra* $\mathcal{M}$.

The robustness of the quantum channel performance under inevitable perturbations of the channel and input state [10] is another kind of stability issues which deserves generalizations to algebraic context, in the spirit of analysis of complementarity in [11]. This is a work in progress.

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