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M. Al–N a t o r[1], **S. Al–N a t o r**[2] (Financial University, Moscow)**. On unreported claims models and beta-model for randomized probability of reported claims.**

In this work we generalize the results of [1] (see also [2]).

Let Y be the number of claims incurred each year and X be the number of claims reported by year end. Then

$$q(x) = \mathbf{E}\,(Y \,|\, X = x)$$

and

$$r(x) = \mathbf{E}\,(Y - X \,|\, X = x) = q(x) - x$$

represent respectively the expected total number of claims, and the expected number of unreported claims, both given that $X = x$.

In [1] some models are introduced, for which $q(x)$ is a linear function.

In this work we propose a group of nonlinear models. For these models, it is assumed that for any given claim the probability p of being reported by year end is a random variable with a beta distribution. These models lead to transcendental functions $q(x)$ and $r(x)$. For this reason, the authors have developed the corresponding simulation models. According to the developed simulation models the models with randomized probability more adequately describe the actual arrival process of insurance claims.

Let us now describe the proposed models

Beta-Binomial model. Suppose that p has a beta distribution with parameters a, b. Let the distribution of X is binomial with parameters y, p. Then

$$\mathbf{P}\,\{X = x \,|\, Y = y) = \frac{y!}{x!(y-x)!}\,\frac{B(x+a, y-x+b)}{B(a,b)}.$$

Beta-Poisson-binomial. Assume in the previous model, that Y has a Poisson distribution with parameter λ. Then

$$q(x) = \sum_{y=x}^{\infty} y\,\frac{\lambda^y}{(y-x)!}\,\frac{\Gamma(y-x+b)}{\Gamma(y+a+b)} \Big/ \sum_{k=x}^{\infty} \frac{\lambda^y}{(k-x)!}\,\frac{\Gamma(k-x+b)}{\Gamma(k+a+b)}.$$

Beta-Negative Binomial model. Suppose that Y has a negative binomial distribution with parameters a, b. Let d be the probability of any given claim to be reported by year end. Assume that d is a random variable with a beta distribution having parameters α, β. Then

$$q(x) = \sum_{y=x}^{\infty} y\,\frac{(y+r-1)!}{(y-x)!}\,\frac{\Gamma(y-x+\beta)}{\Gamma(y+\alpha+\beta)}\,(1-p)^y \Big/ \sum_{k=x}^{\infty} \frac{(k+r-1)!}{(k-x)!}\,\frac{\Gamma(k-x+\beta)}{\Gamma(k+\alpha+\beta)}\,(1-p)^k.$$

Calculations corresponding to these models were performed by making use of the developed simulation models in Maple environment. For Beta-Poisson-binomial model (respectively, Beta-negative binomial model) we construct graphics of $q(x)$ for various parameters λ, a, b (respectively, a, p, α, β). According to our calculations, we can conclude that apparently the considered models lead to nonlinear functions $q(x)$ and $r(x)$.

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