ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 23 МАТЕМАТИКИ Выпуск 2
2016

V. V. Levin, **S. A. Khonov**, **S. Y. Guskov** (Moscow, Bauman Moscow State Technical University (BMSTU); Moscow Institute of Electronics and Mathematics (MIEM HSE); Bauman Moscow State Technical University (BMSTU)).
Maximum likelihood estimator for default rate of the credit portfolio.

Banks must calculate reserves for possible credit portfolio losses in accordance with Basel II requirements [3] by the following formula (1):

$$\text{Reserves} = EAD \cdot PD \cdot LGD, \tag{1}$$

where EAD — the Exposure at Default, PD — Default Probability of credit; LGD (Loss Given at Default) — non-payment of funds by credit when default occurs. Banks usually uses different delinquency indexes for control of default risk level.

There is offered to use the maximum likelihood estimator for samples from the stratified set [1, 2] to estimate the credit portfolio default rate.

Let $t_0 < t_1 < \cdots < t_i < t_N$ are the given calendar date, here the month's last days are considered. Let $V_i(t)$ is a vintage (= set of loans, opened during time period $[t_{i-1}, t_i]$) at the current moment t, and V_i is the vintage $V_i(t)$ at the moment $t = t_i$, $i = 1, 2, \ldots, N$. It is clear that $V_i(t) \cap V_j(t) = \varnothing$, $i \neq j$, $V_i(t) = \varnothing$, if $t < t_i$, $i = 1, 2, \ldots, N$. $\cup_{i=1}^{N} V_i(t)$ is a credit portfolio at moment t.

For vintage $V_i = V_{i,D} \cup V_{i,ND}$, where $V_{i,D}(V_{i,ND})$ is the set of defaulted (non-defaulted) credits in the vintage. Quantity $K(V_{i,D})$ of defaulted credits and quantity $K(V_{i,ND})$ of non-defaulted credits in vintage V_i are unknown, but vintage size $K(V_i) = K(V_{i,D}) + K(V_{i,ND})$ is known. Let β_{it} is the rate of observed defaults in $V_i(t)$ at the moment $t, i = 1, 2, \ldots, N$.

Maximum likelihood estimator $\widehat{\beta}_{it}$ (from [1, 2]) might be used for assessing default rate of a credit portfolio $\bigcup_{i=1}^{N} V_i(t)$ at the moment t.

It is offered the following maximum likelihood estimator of default probability PD_t for the given moment t:

$$\widehat{PD}_t = \left(\sum_{i=1}^{N} \widehat{\beta}_{it} K\big(V_i(t)\big) \right) \Big/ \sum_{i=1}^{N} K\big(V_i(t)\big).$$

REFERENCES

1. *Ivchenko G. I.*, *Khonov S. A.* An asymptotic estimate for stratified finite populations. — Diskrete Math. Appl., 1989, v. 1, is. 3, p. 87—95.
2. *Ivchenko G. I.*, *Khonov S. A.* Statistical estimation of the composition of a finite set. — Diskrete Math. Appl., 1996, v. 8, is. 1, p. 3–40.
3. Basel II: International Convergence of Capital Measurement and Capital Standards, 2006, p. 86.