ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 23 МАТЕМАТИКИ Выпуск 2
2016

N. M. Mezhennaya (Moscow, Bauman Moscow State Technical University). **On the distribution of the number of runs in polynomial sequence controlled by Markov chain.**

Statistical properties of the number of runs are widely used to check and control properties of discrete random sequences (see [1] and the bibliography ibid). Distributions (including multidimensional) of numbers of runs in the sequences of independent random variables are studied in detail (see [1]). The properties of the random variables in Markov sequences and in sequences of m-dependent random variables are also particularly considered (see [1], [2]). Investigation of properties of the number of runs in a sequence controlled by Markov chain is of big practical importance. This sequence can be considered as a hidden Markov chain (see [3]).

Let $\mathbf{Z} = (Z_0, Z_1, Z_2, \ldots, Z_T, \ldots)$ be homogeneous aperiodical irreducible Markov chain (see [4]) with state set $E_M = \{1, 2, \ldots, M\}$ and probabilities

$$\pi_k(i) = \mathbf{P}\{Z_i = k\}, \quad \pi_{kl}^{(m)} = \mathbf{P}\{Z_{t+m} = l | Z_t = k\}, \quad k, l \in E_M.$$

M probability distributions $\{p_a^{(j)}\}$, $a \in A_N$, $j = 1, 2, \ldots, M$, are defined over the set $A_N = \{1, 2, \ldots, N\}$.

We consider the sequence of random variables $X_0, X_1, \ldots, X_T, \ldots$ taking values in the set A_N with probabilities $\mathbf{P}\{X_j = k\} = p_k^{(Z_j)}$, $k \in A_N$, $j \in \mathbf{N}$.

Let $\nu_t^a = I\{X_{t-1} \neq a, \ \ X_t = \cdots = X_{t+s-1} = a\}$ be the indicator of random event which means that run of a's of length at least s ($s \geqslant 1$) begins at moment t (a-run for brevity), $\varsigma_s^a = \sum_{t=1}^{T-s+1} \nu_t^a$ be the number of a-runs with length at least s in the sequence $X_0, \ldots, X_T$.

The mean of random variable ς_s^a is defined by formula $\mathbf{E}\varsigma_s^a = \mathbf{E}\lambda_s^a(\mathbf{Z})$, where

$$\lambda_s^a(\mathbf{Z}) = \mathbf{E}(\varsigma_s^a | \mathbf{Z}) = \sum_{t=1}^{T-s+1} (1 - p_a^{(Z_{t-1})}) \prod_{j=t}^{t+s-1} p_a^{(Z_j)}.$$

Lemma. *Let* $T, s \to \infty$. *Then*

$$\mathbf{E}\,\varsigma_s^a = \lambda_s^a\big(1 + O\left((T-s)^{-1}\right)\big), \tag{1}$$

where

$$\lambda_s^a = (T - s + 1) \sum_{k_0, \ldots, k_s \in E_M} (1 - p_a^{(k_0)}) \widetilde{\pi}_a \prod_{j=1}^{s} p_a^{(k_j)} \pi_{k_{j-1} k_j}.$$

R e m a r k 1. If the initial and stationary distributions of Markov chain are equal then formula (1) has the form $\mathbf{E}\varsigma_s^a = \lambda_s^a$.

Now we turn to the problem of behavior of the distribution of random variable ς_s^a then M is fixed and $T, s \to \infty$. We will use the notation $\mathcal{L}(X)$ for the distribution law of the random variable X, $\mathrm{Pois}(\mu)$ for the Poisson distribution with parameter μ and ρ_{TV} for total variation distance.

Theorem. *Let numbers* M, N *be fixed,* $T, s \to \infty$, $a \in A_N$ *— any fixed character. Then*

$$\rho_{TV}\ (\mathcal{L}(\varsigma_s^a), \mathrm{Pois}\,(\lambda_s^a)) = O\left(s(p_a^*)^s\right),$$

where $p_a^* = \max_{1 \leqslant j \leqslant M} p_a^{(j)}$.

Corollary. *Let conditions of theorem be hold,* $\lambda_s^a \to \widetilde{\lambda}^a \geqslant 0$. *Then* $\varsigma_s^a \xrightarrow{d} Pois(\widetilde{\lambda}^a)$.

R e m a r k 2. It is also possible to prove that for given subset of characters $\mathcal{A} \subset A_N$ and given set of numbers s_a, $a \in \mathcal{A}$, random variables $\varsigma_{s_a}^a$, $a \in \mathcal{A}$, are asymptotically independent and have in the limit Poisson distributions under some additional conditions.

R e m a r k 3. We used local Chen–Stein method to prove theorem (see. [5], [6]).

The work is supported by Russian foundation of basic research (grant 14-01-00318a) and The ministry of education and science of the Russian Federation (project 1.2640.2014).

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