ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
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I. Bugay, **Y. Pastukhova**, **I. Sidorenkova** (Technology University, Korolev Moscow region). **«Soft» mathematic models in ecology.**

The object of research is Baikal seal — endemic of Lake Baikal. Seen in their relations to the general purpose tasks to achieve are to estimate the state of the population and to analyze the dynamics of development depending on the catch. The basis for the present study is provided by statistics from 1970 to the current period. To consider the research of animals in wild nature is extremely difficult. In the early 70 s of the last century a new method for taking into account the population of the Baikal seal has been proposed by professor Dmitriy Pastukhov (Limnological Institute SB of RAS). The method used a specially approach to research Baikal seal population based on calculations from data on the age and sex structure of the population, thus reducing the data error of up to 15–10%.

According to the traditional interpretations of studies in population growth the mathematical model is based on the description of the ecosystem using a differential equation. State of the population can be characterized biomass $m = m(t)$, which we consider a continuously differentiable function of time. Assuming that the growth rate in the small interval of time is directly proportional to the amount of population, we obtain the simplest known Malthus model, describing the dynamics of the species in the "ideal" conditions: $m = km$.

This "hard" model carrying into an exponential growth of the population. For large m there is saturation and "hard" model of the phenomenon should be replaced by a "soft": $m = k(m)m$. In the simplest case, the coefficient is $k(m) = a - bm$. $A = a/b$ is characteristics of the environment, its "capacity". The model is called logistics. The solution of the differential equation is $t = c - \frac{1}{a} \ln |\frac{A}{m} - 1|$. Integral curves shown in Figure 1.

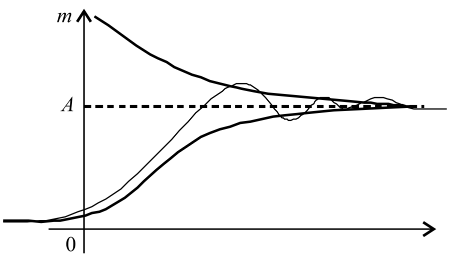

Fig. 1

Evaluation the coefficients a, b using statistical data 1970–1987 years (the data obtained have been report accurately and objectively). Estimation of the parameters a and b are carried out by the differential equation $m = (a - bm)m$ by transformation it to the form: $(\ln m)' = a - bm$. Parameter estimation was obtained by method of least squares: $a = 0,291585$, $b = 0,003132$, $A \approx 93$ (thousands of individuals). The result is a logistic curve of the form $m = \frac{A}{1+Be^{-at}}$, $B - \frac{A}{m_0} - 1$ (Figure 2).

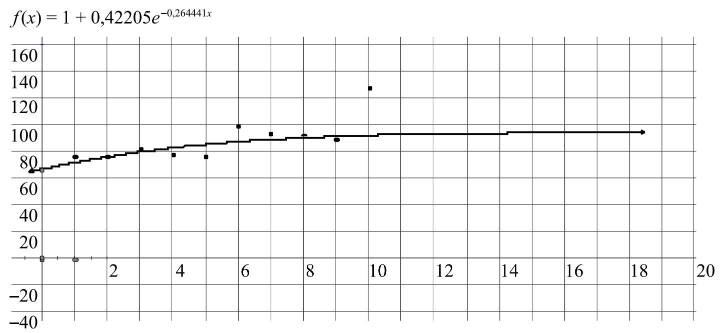

Fig. 2

What is the impact of catch on the population state (q value includes the size of both formal and informal prey)? To answer this question, we consider the equation: $m = b(A-m)m-q$. The critical value $q = \frac{a^2}{4b}$ is determined by the condition $m > 0$. For the study period 1970–1987 $q = 6,787$ (thousands of individuals). Given that $q < q_{\text{кр}}$ system has two stable equilibrium states: stable m_2 and unstable m_1. If due to any reasons the mass falls below the level m_1, in the future population will be wiped out within a finite time (albeit slowly, if mass differ from m_1 small) (Figure 3).

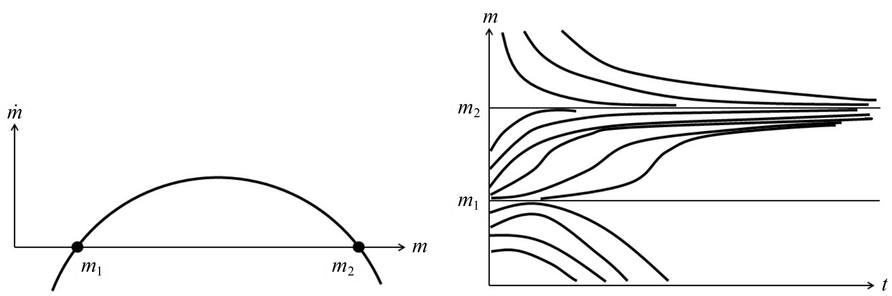

Fig. 3

Given that $q > q_{\text{кр}}$ population is destroyed in a finite time, although she was great at the initial moment (Figure 4).

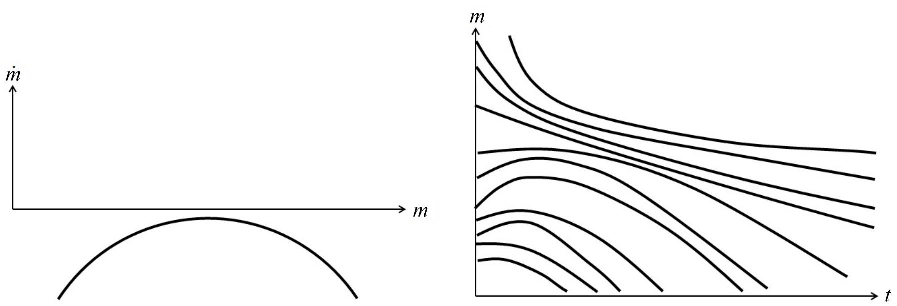

Fig. 4

Because the initial statistics are essential error, then the characteristic of the environment could be somewhat different than the calculated. Moreover, in the opinion of scientists it is not constant and is subject to change under the influence of various factors, not yet studied in full. Therefore the dependence of the critical “quota” from the “capacity” of the lake can be great interest. The solution of the differential equation is transformed into form: $\ln(\frac{A}{m}-1)=\ln B-at$, from which the values of the parameters a and $\ln B$ determined for a given value of A. Thus if $A=130$ we get $q=3,756$ (thous. individuals), with $A=140$ $q=2,831$, with $A=150$ $q=2,552$, with $A=160$ $q=2,416$. It follows that the critical “quota” is inversely proportional to the “capacity” of habitat. The baseline results of investigation permit the following conclusion: to obtain more accurately evaluation of the critical values of catching it is possible shift attention to environmental characteristic as the underestimation of this index leads to overstated values “quota” catch.

Thus, a more precise characterization of the state population is about its weight as compared to the “capacity” of habitat.

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