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E. V. A n d r i a n o v a, V. I. A s t a f e v (Samara, Samara State Technical University). **The influence of discontinuities in reservoirs on the character of oil flow to the production wells.**

In spite of the most oil fields are on the final stage of the field development, there are a lot of ways to maintain and to increase the production. The modern level of science and technology allows extracting oil more efficiently, taking into account the individual characteristics of reservoir and the behavior of fluids.

A part of oil reserves are concentrated in fractured reservoirs [1]. A characteristic feature of the development process of such reservoirs has the deviation in well productivity and rock permeability, significant dependence of IPR curves on the pressure, etc. For fractured reservoirs the main flow of oil to the well occurs through the fracture [2].

Let us consider a plane stationary flow of incompressible fluid with viscosity μ to the vertical production well with well flow rate Q and placed at the point z_0. The flow occurs in an isotropic porous medium with thickness h and permeability k. Inside the external boundary there is a crack with length $2l$ and thickness $2\delta(\delta \ll l)$ and permeability k_f. Suppose the crack is oriented along the x-axis, and its center coincides with the origin of the plane (x, y). As described in paper [3], the flow potential can be represented in the form:

$$\Phi(z) = \frac{2\pi kh}{Q\mu}\varphi(z) = \ln(z - z_0) + \sum_{n=0}^{\infty} c_n z^{-n}. \tag{1}$$

where c_n are unknown coefficients. Because of $\delta \ll l$, it was proposed [4] to replace the ellipse with semi-axes l and δ by straight-line section of zero thickness $(-1 \leqslant \xi = x/l \leqslant 1)$. Then the fluid flow in the fracture can be modeled as the following additional boundary conditions on the cut:

$$\begin{cases} \alpha_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Re}\,(\Phi^+ - \Phi^-) = \mathrm{Im}\,(\Phi^+ - \Phi^-), \\ \beta_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Im}\,(\Phi^+ + \Phi^-) = -\mathrm{Re}\,(\Phi^+ - \Phi^-), \end{cases} \tag{2}$$

where $\alpha_0 = \dfrac{\delta k_f}{lk}$ and $\beta_0 = \dfrac{\delta k}{lk_f}$.

We will look the function $\Phi(z)$ in the form, as $\Phi(z) = \Phi_1(z) + \Phi_2(z)$, where $\mathrm{Re}\Phi_1^+(z) = \mathrm{Re}\Phi_1^-(z)$; $\mathrm{Im}\Phi_1^+(z) = -\mathrm{Im}\Phi_1^-(z)$; $\mathrm{Re}\Phi_2^+(\mathrm{z}) = -\mathrm{Re}\Phi_2^-(\mathrm{z})$; $\mathrm{Im}\Phi_2^+(z) = \mathrm{Im}\Phi_2^-(z)$. Then the boundary conditions (2) will be as:

$$\begin{cases} \alpha_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Re}\,(\Phi_1^+) = \mathrm{Im}\,(\Phi_1^+), \\ \beta_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Im}\,(\Phi_2^+) = -\mathrm{Re}\,(\Phi_2^+), \end{cases} \tag{3}$$

Mapping the exterior of the section $-l < x < l$, $y = 0$ on the exterior of a unit circle $|\nu| = 1$, the potential (1) in a new variable ν can be written as:

$$\Phi(\nu) = \ln(\nu - \nu_0) + \sum_{n=0}^{\infty} a_n \nu^{-n}, \tag{4}$$

where $l\nu(z) = z + \sqrt{z^2 - l^2}$, $l\nu(z_0) = z_0 + \sqrt{z_0^2 - l^2}$, $|\nu| > 1$, a_n is new unknown coefficients. Taking into account that $d/d\theta = i\nu d/d\nu$, $\mathrm{Re}(iz) = -\mathrm{Im}z$, $\mathrm{Im}(iz) = \mathrm{Re}z$, the boundary conditions (3) can be rewritten in the form:

$$\begin{cases} \mathrm{Im}\Big(\alpha_0 \nu \dfrac{d\Phi_1^+}{d\nu} - \Phi_1^+\Big) = 0, \\ \mathrm{Re}\Big(\beta_0 \nu \dfrac{d\Phi_2^+}{d\nu} - \Phi_2^+\Big) = 0. \end{cases} \tag{5}$$

Let us consider $a_n = a_n^{(\alpha)} + ia_n^{(\beta)}$ and find $a_n^{(\alpha)}$ and $a_n^{(\beta)}$ from the conditions (5) as:

$$a_n^{(\alpha)} = \frac{n \cdot a_0 - 1}{n \cdot a_0 + 1} \frac{\cos n\theta}{n\rho_0^n}; \quad a_n^{(\beta)} = \frac{n \cdot \beta_0 - 1}{n \cdot \beta_0 + 1} \frac{\sin n\theta}{n\rho_0^n}; \tag{6}$$

The nature of fluid flow to the wellbore at different locations of the well for different values of the fracture conductivity F_{CD} [5] are shown on the Figure.

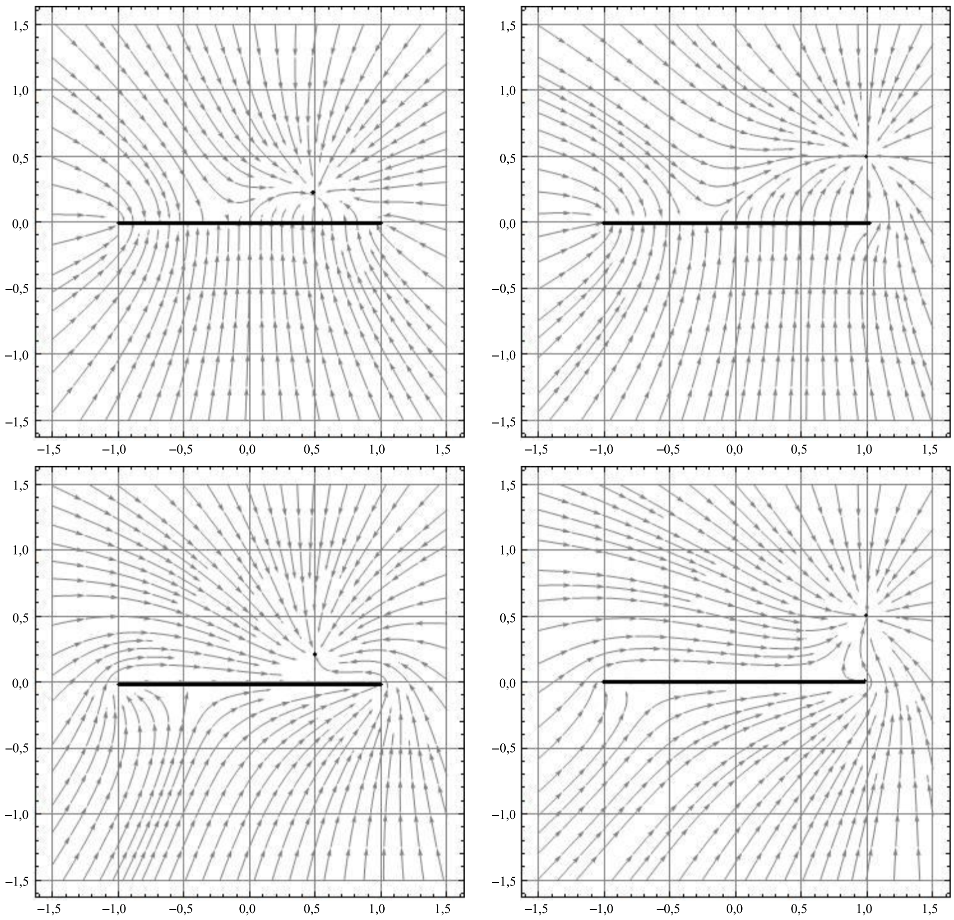

Fig. Streamlines of the fluid flow to the well, located at the points (0.2, 0.5) and (0.5, 1) for the values of $F_{cd} = \infty$ and $F_{cd} = 0$

Conclusion. In this work the formulation and solution of the problem of fluid flow to the well at the presence of a crack of different conductivity FCD has been done. More general boundary conditions was considered taking into account pressure difference above and below the section. For different values of FCD and various well-crack locations the nature of the fluid flow to a well has been analyzed.

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