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V. I. Pagurova, N. S. Chizhikova (Moscow, Lomonosov MSU). **On the joint limiting distribution of central and intermediate order statistics.**

We consider the joint asymptotic distribution of central and intermediate order statistics when a sample size tends to infinity.

Let $X_1, \dots, X_n$ be mutually independent random variables with the common distribution function $F(x)$, $f(x) = F'(x)$, the set of order statistics is $X_1^{(n)} \leqslant \dots \leqslant X_n^{(n)}$. Let $0 < t_1 < \dots < t_m$, $t_{m+2} > \dots > t_{m+l+1} > 0$, $0 < \alpha < 1$, $0 < p < 1$, $[x]$ denotes the integer part of x, $n_i = [t_i n^\alpha]$, $i = 1, \dots, m$, $n_{m+1} = [np] + 1$, $F(\zeta) = p$, $n_j = [n - t_j n^\alpha + 1]$, $j = m+2, \dots, m+l+1$. Define

$$\lambda_{k,n} = k/n, \quad k = k(n) \to \infty, \quad \lambda_{k,n} \to 0, \text{ as } \quad n \to \infty. \tag{1}$$

Necessary and sufficient conditions under which the statistic $T_n = (X_k^{(n)} - d_n)/c_n$ has a Gaussian distribution, as $n \to \infty$, for some $c_n > 0$ and d_n were shown in [1, 2]. If the condition (1) is satisfied then the the Gaussian and log-Gaussian distributions are possible limiting distributions [3, 4]. The joint asymptotic distribution, as $n \to \infty$, for central order statistics of a rank $[n\lambda_i] + 1$, $i = 1, \dots, m$, $0 < \lambda_1 < \dots < \lambda_m < 1$, was given in [5]. The joint asymptotic distribution of intermediate order statistics when random variables $X_1, \dots, X_n$ satisfy some conditions of dependence was shown in [6]. The asymptotic limit distribution of intermediate order statistics based on the sample with a random size was considered in [7].

We introduce variables $d_{n,i}$ and $b_{n,j}$ satisfying the following equations

$$F(d_{n,i}) = t_i/n^{1-\alpha}, \quad i = 1, \dots, m, \quad F(b_{n,j}) = 1 - t_j/n^{1-\alpha}, \quad j = m+2, \dots, m+l+1.$$

Theorem. *Let $f(x)$ be a differentiable in the neighborhood of $d_{n,i}$, ζ, $b_{n,j}$, $f(d_{n,i}) \neq 0$, $i = 1, \dots, m$, $f(b_{n,j}) \neq 0$, $j = m+2, \dots, m+l+1$, and $\lim_{n\to\infty} n^{1-\alpha/2} f(d_{n,1}) \neq 0$, $\lim_{n\to\infty} n^{1-\alpha/2} f(b_{n,m+2}) \neq 0$. Then for every ζ, $f(\zeta) \neq 0$, the joint distribution of random variables*

$$X_{n_1}^{(n)} - d_{n,1}, \dots, X_{n_m}^{(n)} - d_{n,m}, X_{n_{m+1}}^{(n)} - \zeta, \\ X_{n_{m+2}}^{(n)} - b_{n,m+2}, \dots, X_{n_{m+l+1}}^{(n)} - b_{n,m+l+1}, \tag{2}$$

as $n \to \infty$, converges to $(m+l+1)$-variate Gaussian distribution with expectations equal *zero and covariances in the following asymptotic presentations*

$$\operatorname{cov}(X_{n_i}^{(n)}, X_{n_j}^{(n)}) = t_i/(n^{2-\alpha} f(d_{n,i}) f(d_{n,j})) \quad i,j = 1,\ldots,m,\ i \leqslant j,$$
$$\operatorname{cov}(X_{n_i}^{(n)}, X_{n_j}^{(n)}) = t_i/(n^{2-\alpha} f(b_{n,i}) f(b_{n,j})) \quad i,j = m+2,\ldots,m+l+1,\ j \leqslant i,$$
$$\operatorname{cov}(X_{n_i}^{(n)}, X_{n_{m+1}}^{(n)}) = t_i(1-p)/(n^{2-\alpha} f(d_{n,i}) f(\zeta)), \quad i = 1,\ldots,m,$$
$$\operatorname{cov}(X_{n_j}^{(n)}, X_{n_{m+1}}^{(n)}) = t_j p/(n^{2-\alpha} f(b_{n,j}) f(\zeta)), \quad j = m+2,\ldots,m+l+1,$$
$$\operatorname{cov}(X_{n_i}^{(n)}, X_{n_j}^{(n)}) = t_i t_j/(n^{3-2\alpha} f(d_{n,i}) f(b_{n,j})), \quad i = 1,...,m,\ j = m+2,\ldots,m+l+1,$$
$$\mathbf{D}X_{n_{m+1}}^{(n)} = p(1-p)/(nf^2(\zeta)).$$

If covariances tend to zero then random variables (2) are independent asymptotically.

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