

III INTERNATIONAL BALTIC SYMPOSIUM
ON APPLIED AND INDUSTRIAL
MATHEMATICS

M. M. Kruchek (Petrozavodsk, PetrSU). **Modelling financial data using generalised hyperbolic distributions: empirical evidence from MOEX Russia Index.**

Barndorff-Nielsen [1] introduced a family of continuous distributions, named the generalised hyperbolic distributions (GHDs). A probability density function of GHD is given by

$$p(x; \lambda, \alpha, \beta, \delta, \mu) = \frac{(\gamma/\delta)^\lambda}{\sqrt{2\pi}K_\lambda(\delta\gamma)} \cdot \frac{K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (x-\mu)^2})}{(\sqrt{\delta^2 + (x-\mu)^2}/\alpha)^{1/2-\lambda}} e^{\beta(x-\mu)}, \quad x \in \mathbf{R},$$

where $\gamma^2 = \alpha^2 - \beta^2$, K_j is the modified Bessel function of the third kind, with order j . It should also be noted that the domain of the parameters must satisfy the following conditions: if $\lambda > 0$, then $\delta \geq 0, \alpha > |\beta|$; if $\lambda = 0$, then $\delta > 0, \alpha > |\beta|$; if $\lambda < 0$, then $\delta > 0, \alpha \geq |\beta|$. Here μ is a location parameter, δ is a scaling factor, α determines the shape, β determines the skewness, and λ reflects the kurtosis of the GHD. The GHD together with its subclasses (namely, hyperbolic (HYP) ($\lambda = 1$), normal-inverse Gaussian (NIG) ($\lambda = -0,5$), variance-gamma (VG) ($\delta \rightarrow 0$) and GH skew Student's t (GHST) ($\alpha \rightarrow |\beta|$) distributions) proved to fit financial returns more adequately when compared to other distributions like the normal and student t distributions [2].

We study log returns of MOEX Russia Index (former MMVB index) to fit GHDs, test whether the subclasses of GHD adequately describe 1-day, 5-days, 15-days, 25-days log returns and identify optimal model. The data set we consider consists of daily closing price of MOEX Russia Index from Moscow Exchange from 06.01.2016 to 29.12.2017. The Augmented Dickey-Fuller (ADF) test confirms stationarity of our log-return series.

We identify NIG distribution as the most suitable subclass of GHDs for describing 1-day, 5-days and 25-days log returns of MOEX Russia Index and GHST distribution for describing 15-days log returns, using statistical methods such as the likelihood-ratio test and the Akaike information criterion. We use R packages **ghyp**, **tseries** to produce results of the various statistical tests mentioned above. Table below summarizes the results.

	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\mu}$	skewness	kurtosis
1-day log returns	-0,5	0,03	14,44	0,02	0,00	-0,01	1,45
5-days log returns	-0,5	1279,93	934,01	0,17	-0,18	0,18	-0,02
15-days log returns	-8,49	18,09	18,09	0,12	0,03	-0,21	-0,2
25-days log returns	-05	44,33	-8,05	0,08	0,03	-0,24	0,49

СПИСОК ЛИТЕРАТУРЫ

1. *Barndorff-Nielsen O. E.* Exponential decreasing distributions of the logarithm of particle size. — Proc. Roy. Soc. London A, 1977, v. 353, p. 401–419.
2. *Eberlein E., Keller U.* Hyperbolic distributions in finance. — Bernoulli, 1995, v. 1, № 3, p. 281–299.