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A. K. Meleshko (Moscow, Bauman MSTU). **On the number of spanning trees in the labeled block-cactus graphs.**

An *articulation point* in a connected graph is its vertex after removing which with the edges incident to it, the graph becomes disconnected. A *block* is a connected graph without articulation points and also a maximal nontrivial subgraph without articulation points. A *cactus* is a connected graph that have no edges lying on more than one simple cycle [1, p. 93]. All blocks in a cactus are edges or simple cycles. A *block-cactus* graph is a connected graph whose all blocks are either complete graphs or simple cycles [2]. The class of block-cactus graphs is a natural extension of the classes of cacti and block graphs.

In [3] the number of spanning trees is obtained in the labeled cacti. The number of spanning trees characterized reliability of information transmission network represented by a graph [4].

Theorem. *Let* $t(F_n(n_2, n_3, \ldots, m_4 \ldots))$ *be the number of spanning trees in the labeled block-cactus graph with* $n \geqslant 2$ *vertices, having* $n_2 \geqslant 0$ *edge — blocks and* $n_i \geqslant 0$ *polygon — blocks with* i *vertices at* $i \geqslant 3$, $m_i \geqslant 0$ *complete graph blocks with* i *vertices at* $i \geqslant 4$. *Then the formula is valid*

$$t(F_n(n_2, n_3, \ldots, m_4 \ldots)) = \prod_{i\geqslant 3} i^{n_i} \prod_{i\geqslant 4} (i^{i-2})^{m_i},$$

where $n-1 = n_2 + 2n_3 + \sum_{i=4}^{\infty}(i-1)(n_i+m_i)$

Proof. It was proved in [3] that the number of spanning trees in the connected graph is equal to the product of the numbers of spanning trees its blocks. The number of simple cycles in the block-cactus graph is equal to $k = \sum_{i\geqslant 4} \frac{(i-2)(i-1)}{2} m_i + \sum_{i\geqslant 3} n_i$, and the number of its edges is equal to $l = n_2 + \sum_{i\geqslant 3} i n_i + \sum_{i\geqslant 4} \frac{i(i-1)}{2} m_i$. Since $k = l - n + 1$ we obtain $n - 1 = n_2 + 2n_3 + \sum_{i=4}(i-1)(m_i+n_i)$. The blocks of the block-cactus graph are either complete graphs or simple cycles. Since the number of spanning trees in the block graph with n vertices is equal to n^{n-2}, and the number of spanning trees in the simple cycle with n vertices is equal to n, then using the lemma [3] we obtain the statement of the theorem.

As $m_i = 0$ a formula is obtained for the number of spanning trees in the labeled cactus $t(F_n(n_2, n_3, \ldots)) = \prod_{i\geqslant 3} i^{n_i}$, where $n - 1 = n_2 + 2n_3 + \cdots$. This formula was proved in [3], that is naturally, as cacti is a special case of block-cactus graphs.

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