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I. V. Danilova (Petrozavodsk, IAMR KarRC RAS). **Dynamics of population patch distribution.**

The problem of selection by a population of a patch in the absence of information on an utility of a patch, that is, the volume of its energy resources, is considered. Dynamics of population behavior in the case of selection of patch is described by a system of ordinary differential equations with respect to a poulation patch distribution. U. Dieckman [1] proposed an approach to the population patch distribution modeling. This approach is based on a utility function that takes into account the amount of resources in a patch, the distance of a population from a patch, and the measure of an information certainty on a true patch utility. In this case, the Boltzmann distribution is used to describe the population patch distribution. In this paper, the dynamic system that describes the population patch distribution, which depends on the utility of the patch, is proposed. The Boltzmann distribution is a particular solution of the proposed system of differential equations. The Lyapunov stability condition for the Boltzmann distribution is obtained. The utility functions of the patches, which depend on the population — patch distance and the measure of the information certainty, are introduced.

Let assume, that $U_i(d_i)$ is the utility of apatch i from the point of view of the population, which is at a distance $d_i = d_i(t)$ from it, t is a time. In the following [1] utility function is considered: $U_i(d_i) = I(d_i)V_i - T(d_i) + (1 - I(d_i))\overline{V}$, where V_i is the true utility of a patch i, $I(d_i)$ is the measure of information certainty on a patch i true utility, $T(d_i)$ is the cost of moving to patch i, $\overline{V} = \sum_{i=1}^{m} I(d_i)V_i$ is the average measure of information certainty on patches, $d_i = \rho(M, A_i)$ is the distance between the current position of population ($M = M(t) \in R^2$) and a patch (point $A_i \in R^2$). In this paper, the measure of information certainty and the cost of moving are set by the following formulas, respectively: $I(d_i) = e^{-\beta d_i^2}$, $T(d_i) = \alpha d_i^2$, where α and β are positive parameters.

In this paper a concept of a domain D_i of preferred utility for m patches in space R^n is proposed

$$D_i = \{M \in R^n : U_i(\rho(M, A_i)) > U_j(\rho(M, A_j)), \quad i \neq j, \quad j = 1, \ldots, m\}.$$

Domains of preferred utility D_i are constructed for case $m = 2$. Let us note, that if $U_i = -d_i$ is the function of utility, then the domain of preferred utility partion of the plane is the Voronoi diagram.

In this paper, the Boltzmann distribution is used to describe the patch population distribution in the case of moving population:

$$P_i(t) = \frac{e^{qU_i(d_i)}}{\sum_{i=1}^{m} e^{qU_i(d_i)}},$$

where $d_i = d_i(t)$, t — is a time. Via differentiation Boltzmann's distribution according to time, we receive the system of m non-linear non-autonomic ordinary differential equations, which describes the patch distribution dynamics:

$$\begin{cases} \dot{P}_1 = q \cdot P_1 \cdot (P_2 \cdot \varphi_{12}(t) + \cdots + P_m \cdot \varphi_{1m}(t)) \\ \dot{P}_2 = q \cdot P_2 \cdot (P_1 \cdot \varphi_{21}(t) + \cdots + P_m \cdot \varphi_{2m}(t)) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dot{P}_m = q \cdot P_m \cdot (P_1 \cdot \varphi_{m1}(t) + \cdots + P_{m-1} \cdot \varphi_{mm-1}(t)) \end{cases},$$

where P_i is the probability of a patch i selection, $\varphi_{ij}(t) = \dot{U}_i - \dot{U}_j$, q is a positive parameter. The solution of the system was found for case $m = 2$:

$$\begin{cases} \dot{P}_1 = q \cdot P_1 \cdot P_2 \cdot \varphi(t) \\ \dot{P}_2 = -q \cdot P_1 \cdot P_2 \cdot \varphi(t) \end{cases} \tag{1}$$

The following theorem concerning the Lyapunov stability of the Boltzman distribution is proved.

Theorem *Assume* $\varphi(t)$ *is a continuous function. Then the solution of the system* (1), *which is the Boltzmann distribution, is Lyapunov stable.*

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