OP&PM SURVEYS
ON APPLIED AND INDUSTRIAL
Volume 25 MATHEMATICS Issue 1
2018

III INTERNATIONAL BALTIC SYMPOSIUM ON APPLIED AND INDUSTRIAL MATHEMATICS

I. Talib (Lahore, Pakistan, Virtual University of Pakistan). **Development of the new operational matrices of orthogonal polynomials for finding the approximate solutions of fractional order partial differential models involving mixed partial derivative terms of fractional order.**

In fact, the fractional calculus (FC) which is a generalization of integer–order calculus is best suitable for modeling of various real world physical phenomena which have dependence on two parameters: one is the time instance, and the other is the previous time history. Many physical phenomena in Science and Engineering are fractional in nature, for example, the nonlinear oscillations of earthquakes was studied in [4] by means of fractional modeling, the frequency dependent behaviour of certain viscoelastic materials was studied in [5] by establishing a link between the macroscopic behaviour of certain viscoelastic materials and a molecular theories using the empirically developed FC approach, the applications of FC in continuum mechanics was investigated in [6] by generalizing the classical spring-dashpot models to fractional intermediate models of viscoelastic bodies, moreover in the same article the author also studied the application of FC in statistical mechanics by discussing the unsteady motion of a particle in a viscous fluid which led to the origin of fractional hydrodynamic model which indeed is the best way to explain the fractional Brownian motion of the particles, the applications of FC in financial economics was studied in [7] by addressing a very particular case of the economy of Portugal for the period of 1960–2012 to determine its growth of national economics, namely, the gross domestic product, in Psychology, the time variation of humans emotions was analyzed on the basis of fractional models[9, 10], in Biomechanics, fractional models was used to model the electrical activity of the heart [11, 12], in chemical and material engineering the fractional models was used to model the systems consisting of a plug flow reactor and a continuous stirred tank reactor [13, 14], in solid mechanics, the physical phenomenon used to discuss the dynamics of multi-deformable bodies was modeled on the idea of fractional approach [15, 16]. The differential equations involving mixed partial derivative terms appear widely in the existing literature, for example, in the theory of flow through fissured rock [2], in the shearing motion of a fluid of second grade [3], in the model of heat conduction[1]. In this talk, an accurate numerical method based on operational matrices of fractional order derivatives and integrals in the Caputo and Riemann-Liouville senses of orthogonal shifted Lagendre polynomials is presented for studying the approximate solutions for a generalized class of fractional order partial differential models having mixed partial derivative terms. The technique is extended herein to generalized classes of fractional order coupled systems having mixed partial derivatives terms having applications in Biological studies. One salient aspect of this article is the development of a new operational matrix for mixed partial derivatives in the sense of Caputo. Validity of the method is established by comparing our simulated results with literature solutions obtained otherwise, yielding negligible errors. Furthermore, as a result of the comparative study, some results presented in the literature are extended and improved in the investigation herein.

REFERENCES

1. *Lesnic D.* Heat conduction with mixed derivatives. — Internat. J. Comput. Math., 2004, v. 81, p. 971–977.
2. *Barenblatt G. I.*, *Zheltov Iu. P.*, *Kochina I. N.* Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks (strata). — Appl. Math. Mech., 1960, v. 24, p. 852–864. (In Russian.)
3. *Coleman B. D.*, *Noll W.* An approximation theorem for functionals, with application in continuum mechanics. — Arch. Rational Mech. Anal., 1960, v. 6, p. 355–370.
4. *He J. H.* Nonlinear oscillations with fractional derivative and its applications. In: Proceedings of the 1998 International Conference on Vibrating Engineering. China: Dalian, 1998, p. 288–291.
5. *Bagley R. L.*, *Torvik P. J.* A theoretical basis for the application of fractional calculus to viscoelasticity. — J. Rheol., 1983, v. 27, № 3, p. 201–210.
6. *Mainardi F.* Fractional Calculus: Some basic problems in continum and statistical mechanics. In: Fractal and Fractional Calculus in Continuum Mechanics./ Ed. by A. Carpinteri, F. Mainardi, Heidelberg etc.: Springer, 1997, p. 291–348.
7. *Tejado I.*, *Valério D.*, *Valério N.* Fractional calculus in economic growth modeling: The Portuguese case. In: Proceedings of the 2014 International Conference on Fractional Differentiation and its Applications. Italy: Catania, 2014, p. 1–6.
8. *Saadatmandi A.* Bernstein operational matrix of fractional derivatives and its applications. — Appl. Math. Model., 2014, v. 38, p. 1365–1372.
9. *Ahmad W. M.*, *El-Khazalib R.* Fractional order dynamical models of love, Cha. Solit. Fract. 2007, v. 33, p. 1367–1375.
10. *Song L.*, *Xu L.*, *Yang J.* Dynamical models of happiness with fractional order. — Comm. Nonlinear Sci. Numer. Simul., 2010, v. 15, p. 616–628.
11. *Sundnes J.*, *Linea G. T.*, *Mardal K. A.*, *Tveito A.* Multigrid block preconditioning for a coupled system of partial differential equations modeling the electrical activity in the heart. — Comput. Meth. Biomech. Biomed. Eng., 2002, v. 5, p. 397–409.
12. *Sheng W.* Computer Simulation and Modeling of Physical and Biological Processes using Partial Differential Equations. Phd Dissertation. Univ. Kentucky, 2007.
13. *Aksikas I.*, *Fuxman A.*, *Forbes J. F.*, *Winkin J.* LQ control design of a class of hyperbolic PDE systems: Application to fixed-bed reactor. — Automat. J. IFAC, 2009, v. 45, p. 1542–1548.
14. *Moghadam A. A.*, *Aksikas I.*, *Dubljevic S.*, *Forbes J. Fraser* LQ control of coupled hyperbolic PDEs and ODEs: Applications to a CSTR–PFR system. In: Proceedings of the 9th International Symposium on Dynamics and Control of Process Systems (DYCOPS 2010), Leuven, Belgium, 2010, p. 5–7.
15. *R. Katica, Hedrih,* Fractional order hybrid system dynamics. — Proc. Appl. Math. Mech., 2013, v. 13, p. 25–26.
16. *Parthiban V.*, *Balachandran K.* Solutions of system of fractional partial differential equations. — Appl. App. Math., 2013, v. 8, p. 289–304.