I. S o n i n (Москва/Charlotte, ЦЭМИ РАН/ UNC). **The optimal stopping of Markov chain and its application to other probability problems.**

The optimal stopping (OS) of a Markov chain (MC) is a classical problem of stochastic control. The main goal of the talk is to show that this model has broad applications and can serve as building block for more general applied probability models. The second goal is to show that the State Elimination Algorithm (SEA) for OS of MC developed earlier by the author (Sonin (1995, 1999) can be also applied to a wider range of problems of stochastic control.

We discuss these problems in a framework of a general Markov Decision model (MD-M) which for the discrete case is specified by a tuple $M = (X, A(x), P^a, r(x,a), \beta)$, where X is a finite (countable) state space, $A(x)$ is countable action space, $P^a = \{p(x, y|a)\}$ is a transition matrix and $r(x, a)$ is a reward function at state x if the action a is used, β is a discount factor, $0 < \beta \leqq 1$. The goal is to select a strategy maximizing the expected total reward over all possible strategies π, and to find the value function $v(x) = \sup_\pi E_x^\pi \sum_{i=0}^\infty \beta^i r(Z_i, A_i)$, where (Z_i, A_i) is the random process of states and actions. Without loss of generality we can assume that $\beta = 1$ but there is an absorbing state e and the probability of "survival" $\beta(x) \leqq 1$. We denote P_x, E_x the probability measure and expectation after such modification. If an action space $A(x)$ consists of two actions c, "to continue" and s, "to stop", we obtain an OS model. In this case we denote $r(x, c) = c(x)$, a *one step cost function*, $r(x, s) = g(x)$, a *terminal reward function,* $Pf(x) = \sum p(x, y)f(y)$. Then the classical problem of OS of MC is

P r o b l e m 1. To find the value function v, and to find, if it exists, the optimal stopping set $S = \{x : g(x) = v(x)\}$, and thus to solve the corresponding *Bellman* (*optimality*) equation $v = \max(g, c + Pv)$.

The following class of applied probability models is a special case of a general MDM but much more general than OS model. A general CRQ **(continue, restart, quit)** model is a model, where a subset of state space X, $B = \{s_1, \ldots, s_m\}$ is fixed, and at each state x a set of available actions $A(x) = \{c, q, r_j, j = 1, \ldots, m\}$. If an action c, "continue" is selected then $r(x, c) = c(x)$ and transition to a new state occurs according to transition probabilities $p(x, y)$ from a stochastic matrix P, if an action q, "quit" is selected then $r(x, q) = q(x)$ and transition to an absorbing state e occurs with probability one, if an action r_i, "restart to state s_i" is selected then $r(x, r_i) = r_i(x)$ and transition to a state s_i occurs with probability one.

P r o b l e m 2. To find an optimal strategy and the value function $h(x)$ for the CRQ model. Corresponding Bellman (optimality) equation is

$h(x) = \max[c(x) + Ph(x), q(x), \max_i(r_i(x) + h(s_i)]$.

If set $B = \{s\}$, $\beta(x) = \beta, r(x) = 0$ and $q(x) = -\infty$, for all $x \in X$, i. e. there is only one restarting point, discount factor is a constant, restart fee is equal to zero, and quit action is never used then $h(x)$ coincides with an index introduced by Kathehakis and Veinot (1987). Problem 1 is related to Problem 2 for the case of set $B = \{s\}$ but with general functions $r(x)$ and $q(x)$ through the following construction (*Retirement Process* model) introduced by Whittle (1980). Given a model M, let us consider the *parametric family* of OS models $M(k)$, where the terminal reward function $g(x) = r(x) + k$ for all $x \in X$, k is a real number. Denote $v(x, k)$ the value function for such a model and let $w(x) = \inf\{k : v(x, k) = k + r(x)\}$. Generalizing the result of Whittle we prove that $h(x) = w(x)$ and to calculate $w(x)$ we apply SEA. To solve Problem 2 we develop an iterative procedure. The problems under consideration are related closely also to a classical Gittins index and to its generalizations analyzed in Sonin (2008) and Presman, Sonin (2006).