A. Chuprunov, I. Fazekas (Казань, КГУ/ИЭУиП; Дебрецен, Дебреценский ун-т). **Strong law of large numbers for random forests and it's applications.**

Let $T_{n,N}$ be a random forest of ordered trees with N roots and n nonroots vertices. Let $\mathbf{Z}^+$ be a set of nonnegative integer numbers. Denote by $\mathbf{I}_{Nni}^{(r)}$ an indicator of the event: the ith tree has r nonroots vertices. Then $\mu_r(N,n) = \sum_{i=1}^{N} \mathbf{I}_{Nni}^{(r)}$ is a number of trees with r nonroots vertices. Let

$$\alpha = \frac{n}{N}, \quad \lambda = \frac{\alpha}{1+\alpha} \quad \textit{and} \quad L(r,\lambda) = \frac{(1+r)^{r-1}}{r!} e^{-(r+1)\lambda} \lambda^r.$$

Лемма. *Let $\infty > \alpha_1 > \alpha_0 > 0$. Then for $N, n > 2r$ and $\alpha_1 \leqslant \alpha \leqslant \alpha_0$ we have*

$$\mathbf{E}\,(\mu_r(N,n) - \mathbf{E}\,\mu_r(N,n))^4 < CN^2,$$

where C depends from α_0 and α_1 only.

We will assume that all indicators connected with random forests are defined on a same probability space $(\Omega_1, \mathcal{A}_1, \mathbf{P_1})$. Let T_{n_k,N_k}, $k \in \mathbf{N}$ be random forests such that $N_k < N_{k+1}$ for all $k \in \mathbf{N}$. Let $\alpha_k = \frac{n_k}{N_k}$. We will consider

$$S_k = \frac{1}{N_k} \sum_{r \in \mathbf{Z}'} \mu_r(N_k, n_k), \qquad k \in \mathbf{N},$$

where $\mathbf{Z}' \subset \mathbf{Z}^+$.

Теорема 1. *Let $\alpha_k \to \alpha$, as $k \to \infty$. Then we have*

$$S_k \to L, \quad \textit{as} \quad k \to \infty,$$

almost sure, where $L = \sum_{r \in \mathbf{Z}'} L(r,\lambda)$.

Let $T_{n,N}$, $n, N \in \mathbf{N}$ be random forests such that $T_{n+1,N}$ is obtained from $T_{n,N}$ by addition a new vertices. Denote $\mathbf{I}_{Nni}^{(r\infty)} = \sum_{m=r}^{\infty} \mathbf{I}_{Nni}^{(m)}$. Consider the random processes

$$Z_N^{(r\infty)}(t) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{I}_{N[Nt]i}^{(r\infty)}, \qquad t \in \mathbf{R}^+.$$

Теорема 2. *Let $r \in \mathbf{Z}^+$. Denote $f_r(t) = \sum_{m=r}^{\infty} L(m,t)$. Then the have*

$$\sup_{t \in \mathbf{R}^+} |Z_N^{(r\infty)}(t) - f_r(t)| \to 0, \quad \textit{as} \quad N \to \infty,$$

for almost all $\omega_1 \in \Omega_1$.

We use Theorem 2 to prove functional limit theorems. Applications to some models of financial markets are given.