M. I b r a g i m o v, **R. I b r a g i m o v**, **P. K a t t u m a n** (Tashkent, Tashkent State University of Economics, Harvard, Harvard University, Cambridge, Judge Business School, University of Cambridge)**. Emerging Markets and Heavy Tails.**

Numerous studies in economics and finance indicate that distributions of many variables of interest in these fields exhibit deviations from Gaussianity, including those in the form of heavy tails (see, among others, the discussion and reviews in [1, 2, 5, 6], and references therein). In models involving a heavy-tailed risk or return r it is usually assumed that the distribution of r has power tails, so that

$$\mathbf{P}\{|r| > x\} \sim C/x^{\zeta}, \tag{1}$$

$\zeta > 0$, $C > 0$, as $x \to +\infty$. Many recent studies argue that the tail indices ζ in heavy-tailed models (1) typically lie in the interval $\zeta \in (2, 5)$ for financial returns on various stocks and stock indices in developed economies (see the reviews in [2, 5], and references therein). A number of empirical estimates (see [2]) support heavy-tailed distributions (1) with tail indices $\zeta \approx 3$ for financial returns on many stocks and stock indices in developed countries and markets. Tail indices $\zeta \in (2, 4)$ imply, in particular, that the returns have finite variances; however, their fourth moments are infinite.

Let $r_1, r_2, \ldots, r_N$ be a sample from a population satisfying power law (1). Further, let

$$|r|_{(1)} \geqslant |r|_{(2)} \geqslant \cdots \geqslant |r|_{(n)} \tag{2}$$

be decreasingly ordered largest absolute values of observations in the sample. Despite the availability of more sophisticated methods, a popular way to estimate the tail index ζ is still to run the following OLS log-log rank-size regression with $\gamma = 0$:

$$\log ; (t - \gamma) = a - b \log |r|_{(t)}, \tag{3}$$

or, in other words, calling t the rank of an observation, and $|r|_{(t)}$ its size: $\log(\text{Rank} - \gamma) = a - b\log(\text{Size})$. The reason for the popularity of the OLS approaches to tail index estimation is arguably the simplicity and robustness of these methods.

Unfortunately, tail index estimation procedures based on OLS log-log rank-size regressions (3) with $\gamma = 0$ are strongly biased in small samples. The recent study by Gabaix & Ibragimov [3] provides a simple practical remedy for this bias, and argues that, if one wants to use an OLS regression, one should use the Rank $-1/2$, and run $\log(\text{Rank} - 1/2) = a - b\log(\text{Size})$. The shift of $1/2$ is optimal, and reduces the bias to a leading order. The standard error on the Pareto exponent ζ is not the OLS standard error, but is asymptotically $(2/n)^{1/2}\zeta$. Numerical results in [3] further demonstrate the advantage of the proposed approach over the standard OLS estimation procedures (3) with $\gamma = 0$ and indicate that it performs well under deviations from power laws and dependent heavy-tailed processes, including GARCH models.

Emerging economic, financial and foreign exchange markets are likely to be more volatile than their developed counter-parts and subject to more extreme external and internal shocks. It is natural to expect that heavy-tailedness properties will be more pronounced in these markets for exchange rates.

The main goal of this talk is a robust analysis of heavy-tailedness properties in emerging and developing foreign exchange markets in comparison to developed markets. In particular, we focus on the analysis of the hypothesis that heavy-tailedness properties are more pronounced in emerging markets exchange rates. We employ, among others, the above robust tail index inference approaches using the bias-corrected log-log rank-size regressions (3) with the optimal shift $\gamma = 1/2$ and correct standard errors proposed in [2]. The robust tail index estimation methods are applied to large data sets on exchange rates for a number of developed and emerging economies. This is in contrast to previous studies of exchange rates in emerging markets that focus on applications of inference methods based on model-specific parametric maximum likelihood procedures and (semiparametric) Hill's estimators, with a number of contributions providing estimates only for relatively small data sets, with potentially non-robust conclusions.

We find that the tail indices for foreign exchange rates in emerging economies are indeed considerably smaller than in the case of developed markets. In particular, the tail index estimates imply that, in contrast to developed markets, the value of the tail index $\zeta = 2$ is not rejected on commonly used statistical significance levels for foreign exchange rates in the most of emerging economies considered. Thus, the variances may be infinite for these exchange rates. In addition, for most of the foreign exchange rates with finite variances, the third and/or fourth moments are infinite, unlike in the developed case. Pronounced heavy-tailedness with tail indices $\zeta \leqslant 2$ presents a challenge for applications of standard statistical and econometric methods. In particular, as pointed out by Granger & Orr [4] and in a number of more recent studies, many classical approaches to inference based on variances and (auto)correlations such as regression and spectral analysis, least squares methods and autoregressive models may not apply directly in the case of heavy-tailed observations with infinite second or higher moments.

Our results imply that traditional economic and financial models and econometric and statistical methods should be applied with care in heavy-tailed settings that are exhibited by exchange rates in a wide range of emerging and developing markets. This is especially important in the case of the tail indices close to the value $\zeta = 1$ that, in many cases, provides the critical robustness boundary (see [5]) and the threshold value $\zeta = 2$ that necessitates deviations from the usual inference methods. The empirical results also indicate that specifications of several models explaining heavy tails may need to be modified in the case of emerging and developing economies. As discussed in the work, the conclusions further have important implications for economic policy decision making and macroeconomic forecasting.

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