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V. A. Rudenko (CEMI RAS). **Dependent error components in stochastic frontier models.**

We analyze an impact of the dependence degree between the error components on parameters estimation in stochastic frontier model. The calculations are made for the 3-factor models: $R_i = \beta_0 \cdot (L_i)^{\beta_1} \cdot (K_i)^{\beta_2} \cdot (I_i)^{\beta_3} \cdot e^{V_i - U_i}$, where R_i is the overall production of the i-th company, L_i is the labor input, K_i is the physical and financial input, I_i is the intellectual capital input, $i = 1, \ldots, 80$ is the number of companies in the sample. Random variables $V_i \sim N(0; \sigma_V^2)$ and $U_i \sim N^+(\mu; \sigma_U^2)$ can be dependent and have Spearman's correlation coefficient $\rho = \rho(V_i, U_i)$. Table 1 summarizes the results for the following models:

- M_1 is a classical model for stochastic production function where V_i and U_i are assumed to be independent and $U_i \sim N^+(0; \sigma_U^2)$;
- M_r is a model where the dependency between the components is described by Gaussian copula, $V_i \sim N(0; \sigma_V^2)$ and $U_i \sim N^+(\mu; \sigma_U^2)$ can be dependent with a dependence parameter $r \in [-1, 1]$ [1, p. 59–65];
- M_α is a model produced by Frank copula, $V_i \sim N(0; \sigma_V^2)$ and $U_i \sim N^+(\mu; \sigma_U^2)$ can be dependent with a dependence parameter $\alpha \in (-\infty, +\infty) \backslash \{0\}$ [2, p. 3–18].

Description of the value $\rho = \rho(V_i, U_i)$, its estimate $\widehat{\rho} = \widehat{\rho}(V_i, U_i)$, the estimation of coefficient of the efficiency correlation $\widehat{s} = \widehat{s}(T\widehat{E}_i, e^{-U_i})$ and hypotheses H_r can be found in [2, p. 3–18].

Table 1. Results for different values of $\rho(V_i, U_i)$

Values of ρ	$\rho = 0.94$			$\rho = 0.79$			$\rho = 0.16$		
Compared models	M_1	M_r	M_α	M_1	M_r	M_α	M_1	M_r	M_α
Estimates of production factors' parameters									
ln K	0.648	**0.66**	0.661	0.644	0.656	**0.658**	0.806	**0.802**	0.802
ln L	0.227	**0.217**	0.219	0.229	0.222	**0.219**	0.123	**0.125**	0.126
ln I	0.143	**0.144**	0.142	0.172	0.166	**0.167**	0.137	**0.140**	0.139
const	0.94	**0.99**	0.98	1.97	1.99	**2.00**	1.77	**1.78**	1.78
Estimates of error components' parameters									
$\widehat{\mu}$	0	**-0.871**	-0.854	0	0.082	**0.084**	0	**0.029**	0.030
$\widehat{\sigma}_V$	0.134	**0.66**	0.857	0.196	0.528	**0.542**	0.275	**0.267**	0.267
$\widehat{\sigma}_U$	0.507	**0.884**	0.957	0.422	0.375	**0.363**	0.644	**0.646**	0.645
$\widehat{\rho}$	0	**0.966**	0.991	0	0.919	**0.989**	0	**-0.011**	-0.041
$\widehat{S}(T\widehat{E}, e^{-U})$	-0.92	**0.93**	0.97	-0.64	0.66	**0.65**	0.36	**0.36**	0.36
Log-likelihood	-20.78	**-19.59**	-19.75	-20.81	-20.49	**-19.06**	-52.11	**-52.05**	-52.05
$\mathrm{H}_r :$ $\left[\begin{array}{l} \text{r=0} \\ \text{или} \\ \alpha \to 0 \end{array}\right.$		**rej.**	rej.		acc.	**rej.**		**acc.**	acc.

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Conclusions

1. If a ranking problem of companies by technical efficiency level is considered the potential dependence between the error components should be taken into account unless there are proven reasons to support the independence assumption.

2. In order to derive the estimates of technical efficiency one should consider several types of copulas. Even if the parameters' estimates are close the maximum values of the respective likelihood functions can vary significantly.

It is especially important to analyze different copulas if the rank of technical efficiencies calculated under independence assumption is not consistent with the one calculated for a model with dependent components.

3. The production factors' parameters can be assessed by means of classical stochastic frontier model since the carried analysis shows low deviation between the estimates of production factors' parameters in the considered models.

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