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Applying Phase Spline-Analysis to Macroeconomic Dynamics.

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Abstract: The report presents an approach to investigate dynamics of macroeconomic indicators. We use spline interpolation as a tool to smooth indicators time dependence, and then construct the phase portrait of the approximating function. This approach allows revealing local phases of data trend alteration and forerunners of economic crises.

Keywords: macroeconomic dynamics, phase spline-analysis, phase trajectory, phase portrait.

Traditional approaches for time series analyzing and forecasting are used to study macroeconomic dynamics. Commonly used smoothing techniques to identify tendencies are moving averages, spectral decomposition, ARMA or ARIMA models, and regression splines analysis. These methods enable to identify cycles [1], crises [2] or compare changes in indicators across different countries [3].

Our study proposes a nonparametric approach to identifying tendencies in a time series that does not use a priori assumptions concerning dependencies in the data under investigation. We assume that the studied data are the results of measuring a continuous function at discrete times. The approach allows identifying local phases of trend alteration and crises forerunners.

Traditional methods of regression and smoothing are not appropriate for such problems. In this paper, we propose to apply a phase spline analysis, which allows solving the problem by explicitly identifying the function rate. In this case, the function is treated as a smoothed series obtained by solving the global interpolation problem.

The paper examines the GDP of Finland, according to annual data over 2000–2019. Data source: the World Bank [4]. Note that the analyzed data must be homogeneous, i. e. measured in the same units (for example, in constant prices related to a certain year). Measuring data in current prices can implement a distortion in the dependencies. The study uses GDP at 2010 constant prices (illustrated at Fig. 1).

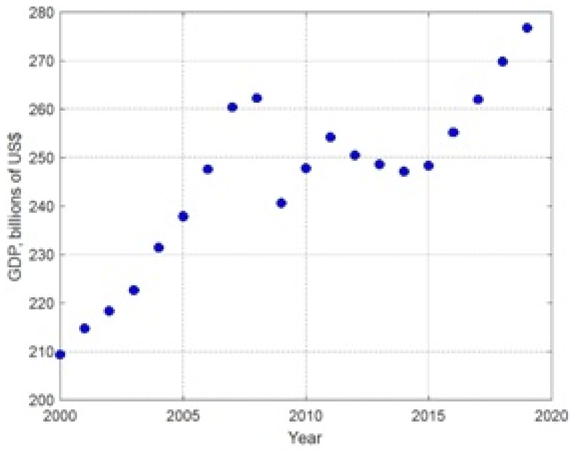

Fig. 1. Finland's GDP over 2000–2019 at 2010 constant prices

The data under study being aggregated and rare (annual), we assume that it does not contain noise and consider the data set as a deterministic grid of a continuous function. Therefore, we can use spline interpolation as a tool to smooth data. The data was interpolated by a natural cubic spline denoted as $y(t)$. Then a phase trajectory was constructed (Fig. 2), which is understood as a trajectory in the (y, y', t) space.

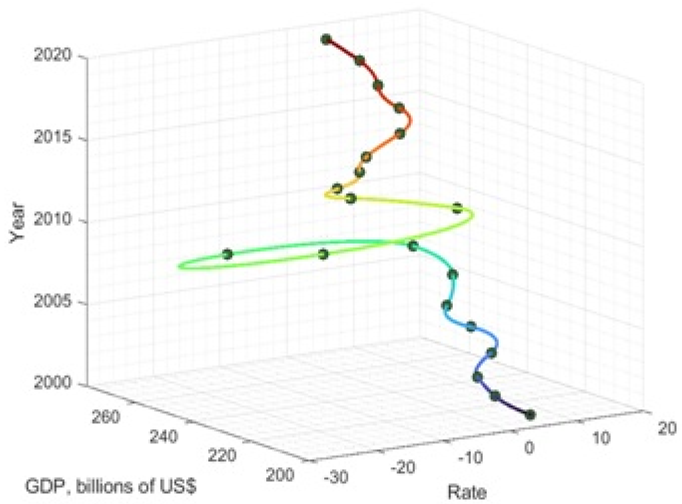

Fig. 2. Phase trajectory of Finland's GDP (spline interpolation)

The study of the phase trajectory projections allows obtaining additional information about the function. Fig. 3 presents the projection onto the plane (y, y') called the phase portrait of the constructed function $y(t)$.

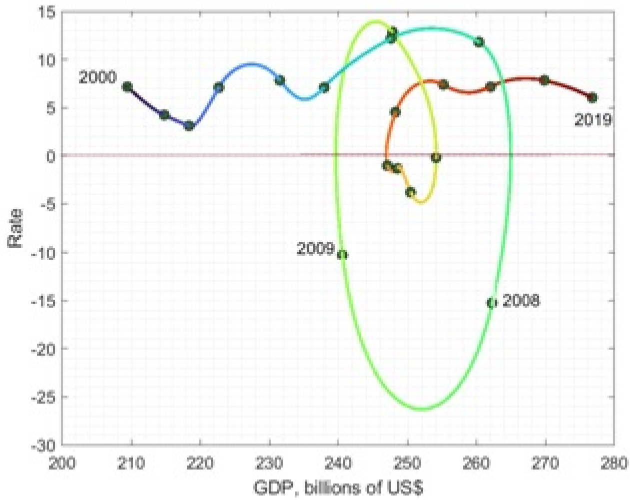

Fig. 3. Phase portrait of Finland's GDP (spline interpolation)

The constructed phase portrait allows us to reveal two economic crises that affected the GDP dynamics of the country. The first one is the 2008–2010 global economic crisis. The second is a smaller-scale crisis — the 2012–2015 financial crisis. There are four stages in the economic cycle popularly known as a full recession, early recovery, late recovery and early recession respectively. We can notice a phase of "rapid" GDP growth before the first crisis, then a slowdown in growth, which was a crisis forerunner, and then the crisis itself. Meanwhile, the data presented at Fig. 1 hardly allows noticing the approach of the crisis. The phase portrait makes this just clear.

The results clearly illustrate the events in the global economy. Applying the proposed method for a set of indicators study allows identifying crises and examine their phases, both a posteriori, and in advance by making forecast based on spline extrapolation of data. Note that the high degree of Finland's integration into the world economy permits us to generalize the findings for other European countries.

The method allows us to identify local deviations from the general trend in the studied series, if there are any. Such deviations may indicate the offensive of a crisis or a breakthrough. In addition, there may be situations when a sharp drop may occur after a positive change in the local trend.

The approach is applicable to high-frequency data with noise. In this case, instead of global spline interpolation, it is advisable to use the method of multidimensional adaptive regression splines (MARS).

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